Outside Options & Risk Attitude

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Motivation

- *Effective risk attitude:* how a decision-maker (DM) chooses among risky prospects (lotteries).
- *Outside options* are important: fundamental part of most choice situations.
- E.g.,
  1. Org choosing projects. *Bankruptcy*.
  2. Manager choosing projects. *Job loss*.
  3. Agent choosing portfolio. *Bankruptcy*.
  4. Health insurance plans. *Public option*.
  5. Occupations. *Retraining*.
  6. Marriage. *Divorce*.
  7. Locations. *Bailout*.

Questions

$$\underbrace{\text{Effective risk attitude}}_{\textit{Observable}} = \underbrace{\text{``True'' risk attitude}}_{\textit{Unobservable}} + \underbrace{\text{Effect of o.o.}}_{\textit{Unobservable}}$$

Q1. Identification. What do choices reveal about "true" risk attitude?

1. Theorem 1a: "true" is *more risk averse* than effective.
2. Theorem 1b: *nothing more* can be learned.

Q2. Comparative statics. How does effective risk attitude vary with "true" & o.o.? Theorem 2.

Q3. Specialness. What transformations must make behavior appear less risk averse?

1. Theorem 1a: *adding an o.o.*
2. Theorem 3: *that's it*.

Literature

Old idea: o.o. increases risk appetite (e.g., Smith ′76).

Newer: Jensen & Meckling (′76), Golbe (′81, ′88), Gollier, Koehl & Rochet (′97).

An important idea, e.g., for financial stability.

Broader literature & connections

Effective risk attitude = "True" risk attitude + Economic forces.

1. *Background risk.* Ross ('81), Kihlstrom, Romer & Williams ('81), Pratt & Zeckhauser ('87), Eeckhoudt, Gollier & Schlesinger ('96), Gollier & Pratt ('96), Pomatto, Strack & Tamuz ('20), Mu, Pomatto, Strack & Tamuz ('24).
2. *(Employment/financing) contracts.* Wilson ('69), Ross ('74), Amihud & Lev ('81), Lambert ('86), Hirshleifer & Suh ('92), Diamond ('98), Garicano & Rayo ('16), Barron, Georgiadis & Swinkels ('20).
3. *Competition.* **R&D:** Dasgupta & Maskin ('87), Klette & de Meza ('86). **Status:** Rosen ('97), Becker, Murphy & Werning ('05), Ray & Robson ('12), Hopkins ('18).

Broader literature & connections

Effective risk attitude = "True" risk attitude + Economic forces.

4. *Career concerns.* Holmström ('82/'99), Hirshleifer & Thakor ('92), Hermalin ('93) & Chen ('15).
5. *Flexibility.* Drèze & Modigliani ('66/'72), Mossin ('69), Spence & Zeckhauser ('72), Machina ('82, '84), Gollier, Lindsey & Zeckhauser ('97), Gollier ('05), Chetty & Szeidl ('07), Postlewaite, Samuelson & Silverman ('08).

Plan (today)

- Setup & background.
- The o.o. model.
- How an o.o. shapes risk attitude. *Proposition 1*.
- **Identification.** *Theorem 1*.
- **Comparative statics.** *Theorem 2*.
- **So what?** *Theorem 3*.
- Further uses?

Setup

- Alternatives $X(\neq \emptyset)$; (simple) lotteries on X.
- EU preferences $\succeq, \succeq'$ on $\Delta(X)$: $\exists\ u : X \to \mathbb{R}$ s.t. $p \succeq q \Leftrightarrow \int u dp \geq \int u dq$.
- u is risk attitude.
- $\alpha u + \beta$ is identified off $\succeq$ ($\alpha > 0$, $\beta \in \mathbb{R}$ unknown).
- u is less risk averse than v iff $\forall\ x \in X$ and $\forall\ p \in \Delta(X)$, $x \succeq (\succ) p \Rightarrow x \succeq' (\succ') p$.

Pratt's Theorem

For $X \neq \emptyset$ and $u, v\colon X \to \mathbb{R}$, the following are equivalent.

- u is less risk averse than v.
- $\exists$ an increasing convex $\phi\colon \mathrm{co}\,(v(X)) \to \mathbb{R}$ that is strictly increasing on $v(X)$ and satisfies $u = \phi \circ v$.
- The following two properties hold:
  1. Order-preserving: $\forall\ x, y \in X\ u(x) \geq (>) u(y) \Rightarrow v(x) \geq (>) v(y)$.
  2. Decreasing secant slope: $\forall\ x, y, z \in X$, if $u(x) < u(y) < u(z)$, then

$$\frac{v(y) - v(x)}{u(y) - u(x)} \geq \frac{v(z) - v(y)}{u(z) - u(y)}.$$

- If X is an open convex subset of $\mathbb{R}$ and u, v C^2 w/ $u', v' > 0$, $\frac{u''}{u'} \geq \frac{v''}{v'}$ "Coefficient of ARA."

The o.o. model

- DM has $v\colon X \to \mathbb{R}$ (vNM expected utility).
- Outside option $K \sim F$ with an *extended CDF* (may put mass at $-\infty$, "unavailable").
- DM's value of $x \in X$:
$$\int \max\{v(x), k\}\, F(dk).$$
- Independence: the inside draw is independent of K.
- DM exercises the o.o. *ex post* if it is better.

Where do o.o.s come from?

Physical o.o. $\sim \mu \in \Delta\big(Y \cup \{\varnothing\}\big)$,
payoff f'n $w : Y \to \mathbb{R}$

'$\varnothing$' means 'unavailable'
convention: $w(\varnothing) := -\infty$

$$F(k) \;:=\; \mu(\{y \in Y \cup \{\varnothing\} : \; w(y) \le k\}) \qquad \forall\, k \in \mathbb{R}. \qquad (\star)$$

Special case: X-valued o.o., i.e. $Y = X$ & $w = v$.

- Implies F concentrated* on $v(X) \cup \{-\infty\}$.
- Conversely, any xCDF F concentrated on $v(X) \cup \{-\infty\}$ arises via $(\star)$ from some X-valued o.o. $\mu \in \Delta\big(X \cup \{-\infty\}\big)$.

*xCDF F is concentrated on S if $\int_S dF = 1$

The o.o. model

Say that (v, F) is an *o.o. representation* of $\succeq$ if

$$p \succeq q \quad \Longleftrightarrow \quad \int \left(\int \max\{v(x), k\} F(dk) \right) dp \geq \int \left(\int \max\{v(x), k\} F(dk) \right) dq.$$

- If (v, F) is an o.o. representation, $\succeq$ is EU w/ risk attitude u: (up to affine translation)

$$u(x) \propto \int \max\{v(x), k\} \, F(dk).$$

- u is effective risk attitude.

How does an o.o. shape risk attitude?

Proposition 1. For $X \neq \emptyset$, $u, v \colon X \to \mathbb{R}$ and xCDF F, the following are equivalent.

1. $\exists \alpha, \beta \in \mathbb{R}$, $\alpha > 0$, s.t.
$$\alpha u(x) + \beta = \int \max\{v(x), k\}\, F(dk) \qquad \forall\, x \in X.$$
2. $\int_0^\infty kF(dk) < \infty$ & $\exists$ a.c. $\phi \colon \mathrm{co}(v(X)) \to \mathbb{R}$ s.t $u = \phi \circ v$, and $\phi' \propto F$ a.e.

When i. everything is money, ii. open & convex $X \subseteq \mathbb{R}$, iii. u, v are C^2 w/ $u', v' > 0$, and iv. $F = G \circ v^{-1}$ for xCDF G

$$2 \quad\Longleftrightarrow\quad \frac{u''}{u'} = \frac{v''}{v'} + \overbrace{\frac{G'}{G}}^{\text{reverse hazard rate!}}$$

Why Proposition 1 is true

- Take $v \equiv v(x)$. O.o.: $v \mapsto \int \max\{v, k\}\, F(dk) =: \phi(v)$.
- Marginal increase of v helps only when o.o. is worse, i.e., RV $K \leq v \Rightarrow$ $\phi'(v) \;=\; \int \mathbf{1}_{\{k \leq v\}} F(dk) \;=\; F(v) \quad \text{(a.e.)}$.
- F is nondecreasing $\Rightarrow \phi$ is increasing and convex.

Proposition 1 sketch $\big($outside option $\Rightarrow$ convex transform$\big)$

From $u(x) \propto \int \max\{v(x), k\} F(dk)$ WTS $u = \phi \circ v$ with $\phi' = \lambda F$ (a.e.).

Define $\phi(v) := \int \max\{v, k\} F(dk)$. Differentiate under the integral $\Rightarrow$

$$\phi'(v) = \int \partial_v \max\{v, k\} F(dk) = \int \mathbf{1}_{\{k \le v\}} F(dk) = F(v) \quad \text{(a.e.)}.$$

Proposition 1 sketch $\big($convex transform $\Rightarrow$ outside option$\big)$

Assume $u = \phi \circ v$ with ϕ increasing, convex, absolutely continuous, and $\phi' = \lambda F$ a.e. for some $\lambda > 0$ (plus $\int_{(0,\infty)} k\, F(dk) < +\infty$).

Let $H(v) := \int \max\{v, k\}\, F(dk)$. Then as above $H'(v) = F(v)$ (a.e.) and

$$H(v) = H(0) + \int_0^v F(t)\, dt, \quad \text{and} \quad \phi(v) = \phi(0) + \lambda \int_0^v F(t)\, dt.$$

Thus, $\phi(v) = \lambda\, H(v) + \big(\phi(0) - \lambda\, H(0)\big)$. Setting $\alpha := \frac{1}{\lambda}$ and $\beta := H(0) - \frac{1}{\lambda}\phi(0)$ gives

$$\alpha\, u(x) + \beta = H(v(x)) = \int \max\{v(x), k\}\, F(dk).$$

What do choices reveal about "true" risk attitude?

Theorem 1. Suppose $u, v \colon X \to \mathbb{R}$ are bounded above and u is Lipschitz with respect to v. The following are equivalent:

1. u is less risk-averse than v.
2. There exist $\alpha > 0$, $\beta \in \mathbb{R}$, and an xCDF F (with $F > 0$ on $(\inf v(X), \infty)$) s.t.

$$\alpha\, u(x) + \beta \;=\; \int \max\{v(x), k\}\, F(dk) \qquad \forall\, x \in X.$$

$2 \Rightarrow 1$ *Positive:* effective risk 'tude u is a lower-bound on "true" 'tude v

$1 \Rightarrow 2$ *Negative:* nothing more can be learned about v from u.

Theorem 1

Proposition 1: o.o.s act as increasing, convex transformations of v and reduce risk aversion.

Theorem 1: an equivalence. Every admissible “flattening” of v is exactly “add an outside option.”

Theorem 1 sketch

$(1 \implies 2)$

1. **Start from what "less RA" means.** By Pratt/Yaari, there is an increasing, convex ψ with $u = \psi \circ v$.
2. **Look at slopes.** Convexity $\Rightarrow$ the right derivative ψ'_+ exists and is nondecreasing; Lipschitz of u in $v \Rightarrow \psi'_+$ is bounded. Normalize a scale $\alpha > 0$ and define
$$F(v) := \alpha\psi'_+(v).$$
   Then F is nondecreasing, right-continuous, $0 \leq F \leq 1$: an xCDF!
3. **Integrate the slope.** Recover ψ by
$$\psi(v) = \psi(0) + \frac{1}{\alpha}\int_0^v F(t)\,dt.$$

Theorem 1 sketch (II)

4. **Identify the "outside-option."** For any xCDF F, set $H(v) := \int \max\{v, k\} F(dk)$. Then $H'(v) = F(v)$ (a.e.) and, anchoring at 0,

$$H(v) = H(0) + \int_0^v F(t)\, dt.$$

   Thus $\psi(v) = \frac{1}{\alpha} H(v) + \psi(0) - \frac{1}{\alpha} H(0)$.

5. **Compose back and tidy constants.** Since $u = \psi \circ v$, with $v = v(x)$,

$$\alpha\, u(x) + \beta = H(v(x)) = \int \max\{v(x), k\}\, F(dk),$$

   where $\beta := H(0) - \alpha\psi(0)$.

$(1 \Longleftarrow 2)$ **Easy direction (already in Prop. 1).** Given an o.o. with $F > 0$ on $(\inf v(X), \infty)$, Prop. 1 gives $u = \psi \circ v$ with ψ increasing and convex, and $F > 0$ ensures ψ is strictly increasing on $v(X)$; hence, u is less RA than v.

What do choices reveal about o.o.?

Proposition 2. For u and xCDF F, the following are equivalent

1. $\int_{(0,+\infty)} kF(dk) < +\infty$ and if $\int_{(-\infty,0]} kF(dk) > -\infty$ then u is bounded below.
2. $\exists \alpha, \beta \in \mathbb{R}$, $\alpha > 0$, and $v \colon X \to \mathbb{R}$ s.t.

$$\alpha u(x) + \beta = \phi(u(x)) = \int \max\{v(x), k\}\, F(dk) \qquad \forall\, x \in X.$$

V. negative: observation of ′tude u tells us nothing about F except that $\int_{(0,+\infty)} kF(dk) < +\infty$ and, if u is unbounded below, that $\int_{(-\infty,0]} kF(dk) = -\infty$.

Proposition 2

- **Non-identification of the fallback.** From observed effective attitude u alone, the outside-option distribution F is *not* identified beyond tail-integrability facts. *Many F can rationalize the same u.*
- **Re-labeling invariance.** For any F satisfying the tail conditions, one can choose $\alpha > 0, \beta \in \mathbb{R}$ and define v so that

$$\alpha\, u(x) + \beta \;=\; \int \max\{v(x), k\}\, F(dk),$$

  i.e., different F are observationally equivalent once v is re-parameterized.
- **What is identified.** Theorem 1 implies only that the *true* attitude v is (weakly) more risk-averse than u; there is no sharper information about F without extra structure or variation.

Comparative Statics Preliminary

Given $K \subseteq \mathbb{R}$ xCdfs F and $\hat{F}$, say that $\hat{F}$ is better than F in the reverse hazard rate order on K if and only if

$$F(\ell)\hat{F}(k) \leq F(k)\hat{F}(\ell) \quad \forall\ k, l \in K,\ k < l$$

If K is an open interval and $F, \hat{F}$ are C^1 w/ $F, \hat{F} > 0$ on $(\inf K, +\infty)$, equivalent to $\hat{F}'/\hat{F} \geq F'/F$ on K.

Comparative Statics

Theorem 2. Fix outside-option representations

$$\alpha u(x)+\beta=\int \max\{v(x),k\}F(dk), \qquad \hat{\alpha}\,\hat{u}(x)+\hat{\beta}=\int \max\{\hat{v}(x),k\}\hat{F}(dk),$$

with F concentrated on $v(X)\cup\{-\infty\}$ and $\hat{F}$ on $\hat{v}(X)\cup\{-\infty\}$. Then:

- Holding v fixed: $\hat{u}$ is less RA than $u \Leftrightarrow \hat{F}\geq_{\mathrm{RHR}} F$ on $v(X)\setminus\{\sup v(X)\}$.
- Holding the *physical* outside-option value distribution fixed (i.e. $F\circ v=\hat{F}\circ\hat{v}$): $\hat{u}$ is less RA than $u \Leftrightarrow \hat{v}$ is less RA than v.

Smooth Special Case

Assume $X \subset \mathbb{R}$ is open and convex, $v, \hat{v} \in C^2$ with $v', \hat{v}' > 0$, and $F, \hat{F} \in C^1$. Let $G := F \circ v$ and $\hat{G} := \hat{F} \circ \hat{v}$. Then

$$\frac{u''}{u'} = \frac{v''}{v'} + \frac{G'}{G}, \quad \& \quad \frac{\hat{u}''}{\hat{u}'} = \frac{\hat{v}''}{\hat{v}'} + \frac{\hat{G}'}{\hat{G}}.$$

- If $v = \hat{v}$: $\hat{u}$ is less RA than $u \Leftrightarrow \hat{G}'/\hat{G} \geq G'/G$ (equivalently, $\hat{F} \geq_{\mathrm{RHR}} F$ on $v(X) \setminus \{\sup v(X)\}$).
- If $G = \hat{G}$: $\hat{u}$ is less RA than $u \Leftrightarrow \hat{v}$ is less RA than v.

Recall *Likelihood-ratio dominance* $\Rightarrow$ *RHR dominance* $\Rightarrow$ *FOSD*.

What's special (if anything) about adding an o.o.?

Adding an o.o. $\Rightarrow$ transforms problem by replacing each $x \in X$ w/ an x-contingent lottery.

There are other transformations...

E.g., adding background risk?

Are there other transformations that decrease risk aversion?

What's special (if anything) about adding an o.o.?

Formally, replace each x w/ draw from measure μ_x on X.

"True" v versus effective via $x \mapsto \int v d\mu_x$

Which families $(G_x)_{x \in X}$ of CDFs concentrated on X have the property that $\forall$ strictly increasing $v \colon X \to \mathbb{R}$, $x \mapsto \int v dG_x$ is less risk-averse than v?

What's special about o.o.s?

Theorem 3. Let $X \subset \mathbb{R}$ be Borel w/ $\sup X \notin X$, and let $\{G_x\}_{x \in X}$ be CDFs on X. The following are equivalent:

1. $\forall$ bounded, strictly increasing $v\colon X \to \mathbb{R}$, $x \mapsto \int v \, dG_x$ is less RA than v.
2. There exists an xCDF G concentrated on $X \cup \{-\infty\}$ with $G > 0$ on $(\inf X, \infty)$ s.t. for every bounded, strictly increasing v, there exist $\alpha > 0, \beta \in \mathbb{R}$ s.t.

$$\alpha \int v \, dG_x + \beta = \int \max\{v(x), v(y)\} \, G(dy) \quad \forall x \in X,$$

with the convention $v(-\infty) := -\infty$.

Intuition

- The uniform requirement "less RA for *all* increasing v" forbids using the specific curvature of v; the transform must act only via the *value* $v = v(x)$.
- O.o.s do exactly that: $\phi(v) = \mathbb{E}[\max\{v, K\}]$ has slope $\phi'(v) = \Pr[K \leq v]$, an xCDF depending only on v.
- In contrast, background risk or ad hoc perturbations interact with v''/v'; they can raise or lower RA depending on v, so they cannot pass the "for all v" test.

$(2 \implies 1)$ By Theorem 1.

Proof intuition (1 implies 2)

Push G_x forward by v to get a measure $\mu_{v,x}$ on $\mathbb{R}$ (distrb. of RV $v(Y)$ when $Y \sim G_x$). Then, Theorem 1 $\Rightarrow \exists\ \alpha_v > 0$, β_v, and xCDF F_v on values w/

$$\alpha_v \int t\, \mu_{v,x}(dt) + \beta_v \;=\; \int \max\{v(x), t\}\, F_v(dt) \quad \forall\ x \in X.$$

$\Rightarrow$ For this v the rule looks like an outside-option transform *in value space*.

Crucially, $x \mapsto G_x$ is fixed; only v varies.

Hard part: Can find step/affine vs that reorder values in controlled ways. If F_v changed with v, force incompatible representations of the same $\int v(y) G_x(dy)$.

$\Rightarrow$ the outside measure in value space can be chosen the same for all v: $F_v \equiv \mu$.

Proof intuition (1 implies 2) II

$$\alpha_v \int t\, \mu_{v,x}(dt) + \beta_v = \int \max\{v(x), t\}\, \mu(dt) \quad \forall\ x \in X$$

reveals: (up to affine normalization) the effective utility $x \mapsto \int t\, \mu_{v,x}(dt)$ is a common o.o. transformation of $v(x)$.

$\nu \mapsto \int \max\{\nu, t\}\, \mu(dt)$ determines μ (a.e. via its derivative $\mu((-\infty, \nu]))$.

As v is strictly increasing, choose Q on $X \cup \{-\infty\}$ with pushforward $v_\# Q = \mu$. Then

$$\alpha_v \int v(y)\, G_x(dy) + \beta_v = \int \max\{v(x), v(y)\}\, Q(dy),$$

which is exactly "max with a fixed outside draw" (up to affine normalization).

Conclude "$\forall\ v$" collapses transformations to *max w/ a fixed outside draw*.

One use

Weinstein ('16), Battigalli et al. ('16): decreased risk aversion expands rationalizable (undominated) set.

Curello et al. ('25): a converse. Universal expansions $\iff$ greater risk propensity!

Combine: model is lower bound on what can be rationalized!

Actions not purely dominated $\equiv$ actions rationalized by some utility. This is the set of actions that can be rationalized by o.o.!

Thank you!

Still here? Ok, another use

Pease & Whitmeyer ('23) "Playing it Safe: Actions Attractive to the Risk Averse"

What makes one action (act) "riskier" than another?

SEU world: state-space Θ; DM w/ a set of actions $A \subseteq \mathbb{R}^{\Theta}$, each continuous (and none dominated by another); utility $u \colon \mathbb{R} \to \mathbb{R}$.

a is *safer* than b, $a \succeq_S b$, if $\forall$ concave & strictly-inc. ϕ, strictly inc. u, and $\mu \in \Delta(\Theta)$,

$$\mathbb{E}_{\mu} u(a_{\theta}) \geq \mathbb{E}_{\mu} u(b_{\theta}) \quad \Longrightarrow \quad \mathbb{E}_{\mu} \phi \circ u(a_{\theta}) \geq \mathbb{E}_{\mu} \phi \circ u(b_{\theta}).$$

PW main theorem

Define

$$\mathcal{A} := \{\theta \in \Theta : a_\theta > b_\theta\} \quad \text{and} \quad \mathcal{B} := \{\theta \in \Theta : a_\theta < b_\theta\}.$$

PW Theorem. The following are equivalent:

1. $a \geq_S b$.
2. For any pair $(\theta, \theta') \in \mathcal{A} \times \mathcal{B}$, $b_{\theta'} \geq a_\theta$ and $a_{\theta'} \geq b_\theta$.
3. For all $\mu \in \Delta$, $F_a^\mu \geq_{sc} F_b^\mu$.
4. For all $\mu \in \Delta$, if $\mathbb{E}_\mu a_\theta \geq \mathbb{E}_\mu b_\theta$, then there exists a lottery $\widetilde{X}$ such that X_a^μ FOSD $\widetilde{X}$ and $\widetilde{X}$ SOSD X_b^μ. Moreover, if $\mathbb{E}_\mu a_\theta = \mathbb{E}_\mu b_\theta$, then X_a^μ SOSD X_b^μ.

Ex I. Contribution games

1. Voting, whistleblowing, public good provision, revolts.
2. Players' payoffs depend on how many others contribute.
3. Anonymous game w/ $n \geq 2$ players. Each player chooses an action $a_i \in \{0, 1\}$.
4. Concretely, take whistleblowing (w/ no exog randomness):
   4.1 At least 1 pl reports, all get G.
   4.2 No one: all get $B < G$.
   4.3 Reporter incurs private cost $c \in (0, G - B)$.
   4.4 $1 \geq_S 0$.
   4.5 Can show: decreased risk aversion $\Rightarrow$ maximal (symmetric) eq reporting probability decreases!
5. Costly majority voting: not voting is safer. O.o$\Rightarrow$ fewer vote!

Ex II. Security demand

State $\theta \in [0,1]$, security pays $S(\theta)$ to an investor. E.g.,

1. Equity: the investor receives a constant portion, $\eta \in (0,1)$, of the cash flow, or $S(\theta) = \eta\theta$,
2. Debt: the investor is owed a debt $d \in (0,1)$ and collects as much of it as possible, or $S(\theta) = \min\{\theta, d\}$; and
3. Call Option: the investor gets a call option with a strike price of $\rho \in (0,1)$, or $S(\theta) = \max\{\theta - \rho, 0\}$.

General rank: $S_a \geq_S S_b$ if and only if $S_b \geq_{sc} S_a$.

$\Rightarrow$ Debt safer than all, call option least safe!

O.o. $\Rightarrow$ demand shifts predictably!

Thank you!