

Outside Options & Risk Attitude

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Motivation

- ▶ *Effective risk attitude*: how a decision-maker (DM) chooses among risky prospects (lotteries).
- ▶ *Outside options* are important: fundamental part of most choice situations.
- ▶ E.g.,
 1. Org choosing projects. *Bankruptcy.*
 2. Manager choosing projects. *Job loss.*
 3. Agent choosing portfolio. *Bankruptcy.*
 4. Health insurance plans. *Public option.*
 5. Occupations. *Retraining.*
 6. Marriage. *Divorce.*
 7. Locations. *Bailout.*

Questions

$$\underbrace{\text{Effective risk attitude}}_{\text{Observable}} = \underbrace{\text{"True" risk attitude}}_{\text{Unobservable}} + \underbrace{\text{Effect of o.o.}}_{\text{Unobservable}}$$

Q1. Identification. What do choices reveal about “true” risk attitude?

1. Theorem 1a: “true” is *more risk averse* than effective.
2. Theorem 1b: *nothing more* can be learned.

Q2. Comparative statics. How does effective risk attitude vary with “true” & o.o.? Theorem 2.

Q3. Specialness. What transformations must make behavior appear less risk averse?

1. Theorem 1a: *adding an o.o.*
2. Theorem 3: *that's it.*

Literature

Old idea: o.o. increases risk appetite (e.g., Smith '76).

Newer: Jensen & Meckling ('76), Golbe ('81, '88), Gollier, Koehl & Rochet ('97).

An important idea, e.g., for financial stability.

Broader literature & connections

Effective risk attitude = “True” risk attitude + Economic forces.

1. *Background risk*. Ross ('81), Kihlstrom, Romer & Williams ('81), Pratt & Zeckhauser ('87), Eeckhoudt, Gollier & Schlesinger ('96), Gollier & Pratt ('96), Pomatto, Strack & Tamuz ('20), Mu, Pomatto, Strack & Tamuz ('24).
2. *(Employment/financing) contracts*. Wilson ('69), Ross ('74), Amihud & Lev ('81), Lambert ('86), Hirshleifer & Suh ('92), Diamond ('98), Garicano & Rayo ('16), Barron, Georgiadis & Swinkels ('20).
3. *Competition*. **R&D**: Dasgupta & Maskin ('87), Klette & de Meza ('86).
Status: Rosen ('97), Becker, Murphy & Werning ('05), Ray & Robson ('12), Hopkins ('18).

Broader literature & connections

Effective risk attitude = “True” risk attitude + Economic forces.

4. *Career concerns*. Holmström ('82/'99), Hirshleifer & Thakor ('92), Hermalin ('93) & Chen ('15).
5. *Flexibility*. Drèze & Modigliani ('66/'72), Mossin ('69), Spence & Zeckhauser ('72), Machina ('82, '84), Gollier, Lindsey & Zeckhauser ('97), Gollier ('05), Chetty & Szeidl ('07), Postlewaite, Samuelson & Silverman ('08).

Plan (today)

- ▶ Setup & background.
- ▶ The o.o. model.
- ▶ How an o.o. shapes risk attitude. *Proposition 1.*
- ▶ Identification. *Theorem 1.*
- ▶ Comparative statics. *Theorem 2.*
- ▶ So what? *Theorem 3.*
- ▶ Further uses?

Setup

- ▶ Alternatives $X(\neq \emptyset)$; (simple) lotteries on X .
- ▶ EU preferences \succeq, \succeq' on $\Delta(X)$: $\exists u: X \rightarrow \mathbb{R}$ s.t. $p \succeq q \Leftrightarrow \int u dp \geq \int u dq$.
- ▶ u is risk attitude.
- ▶ $\alpha u + \beta$ is identified off \succeq ($\alpha > 0, \beta \in \mathbb{R}$ unknown).
- ▶ u is less risk averse than v iff $\forall x \in X$ and $\forall p \in \Delta(X), x \succeq (>)p \Rightarrow x \succeq' (>')p$.

Pratt's Theorem

For $X \neq \emptyset$ and $u, v: X \rightarrow \mathbb{R}$, the following are equivalent.

- ▶ u is less risk averse than v .
- ▶ \exists an increasing convex $\phi: \text{co}(v(X)) \rightarrow \mathbb{R}$ that is strictly increasing on $v(X)$ and satisfies $u = \phi \circ v$.
- ▶ The following two properties hold:
 1. Order-preserving: $\forall x, y \in X \ u(x) \geq (>)u(y) \Rightarrow v(x) \geq (>)v(y)$.
 2. Decreasing secant slope: $\forall x, y, z \in X$, if $u(x) < u(y) < u(z)$, then

$$\frac{v(y) - v(x)}{u(y) - u(x)} \geq \frac{v(z) - v(y)}{u(z) - u(y)}.$$

- ▶ If X is an open convex subset of \mathbb{R} and $u, v \in C^2$ w/ $u', v' > 0$, $\frac{u''}{u'} \geq \frac{v''}{v'}$
"Coefficient of ARA."

The o.o. model

- ▶ DM has $v: X \rightarrow \mathbb{R}$ (vNM expected utility).
- ▶ Outside option $K \sim F$ with an *extended CDF* (may put mass at $-\infty$, “unavailable”).

- ▶ DM's value of $x \in X$:

$$\int \max\{v(x), k\} F(dk).$$

- ▶ Independence: the inside draw is independent of K .
- ▶ DM exercises the o.o. *ex post* if it is better.

Where do o.o.s come from?

Physical o.o. $\sim \mu \in \Delta(Y \cup \{\emptyset\})$,
payoff f'n $w: Y \rightarrow \mathbb{R}$

' \emptyset ' means 'unavailable'
convention: $w(\emptyset) := -\infty$

$$F(k) := \mu(\{y \in Y \cup \{\emptyset\} : w(y) \leq k\}) \quad \forall k \in \mathbb{R}. \quad (\star)$$

Special case: X -valued o.o., i.e. $Y = X$ & $w = v$.

- ▶ Implies F concentrated* on $v(X) \cup \{-\infty\}$.
- ▶ Conversely, any xCDF F concentrated on $v(X) \cup \{-\infty\}$ arises via (\star) from some X -valued o.o. $\mu \in \Delta(X \cup \{-\infty\})$.

*xCDF F is concentrated on S if $\int_S dF = 1$

The o.o. model

Say that (v, F) is an *o.o. representation* of \succeq if

$$p \succeq q \iff \int \left(\int \max\{v(x), k\} F(dk) \right) dp \geq \int \left(\int \max\{v(x), k\} F(dk) \right) dq.$$

- ▶ If (v, F) is an o.o. representation, \succeq is EU w/ risk attitude u : (up to affine translation)

$$u(x) \propto \int \max\{v(x), k\} F(dk).$$

- ▶ u is effective risk attitude.

How does an o.o. shape risk attitude?

Proposition 1. For $X \neq \emptyset$, $u, v: X \rightarrow \mathbb{R}$ and xCDF F , the following are equivalent.

1. $\exists \alpha, \beta \in \mathbb{R}, \alpha > 0$, s.t.

$$\alpha u(x) + \beta = \int \max\{v(x), k\} F(dk) \quad \forall x \in X.$$

2. $\int_0^\infty kF(dk) < \infty$ & \exists a.c. $\phi: \text{co}(v(X)) \rightarrow \mathbb{R}$ s.t. $u = \phi \circ v$, and $\phi' \propto F$ a.e.

When i. everything is money, ii. open & convex $X \subseteq \mathbb{R}$, iii. u, v are C^2 w/ $u', v' > 0$, and iv. $F = G \circ v^{-1}$ for xCDF G

$$2 \iff \frac{u''}{u'} = \frac{v''}{v'} + \overbrace{\frac{G'}{G}}^{\text{reverse hazard rate!}}$$

Why Proposition 1 is true

- ▶ Take $v \equiv v(x)$. O.o.: $v \mapsto \int \max\{v, k\} F(dk) =: \phi(v)$.
- ▶ Marginal increase of v helps only when o.o. is worse, i.e., $\text{RV } K \leq v \Rightarrow \phi'(v) = \int \mathbf{1}_{\{k \leq v\}} F(dk) = F(v)$ (a.e.).
- ▶ F is nondecreasing $\Rightarrow \phi$ is increasing and convex.

What do choices reveal about “true” risk attitude?

Theorem 1. Suppose $u, v: X \rightarrow \mathbb{R}$ are bounded above and u is Lipschitz with respect to v . The following are equivalent:

1. u is less risk-averse than v .
2. There exist $\alpha > 0, \beta \in \mathbb{R}$, and an xCDF F (with $F > 0$ on $(\inf v(X), \infty)$) s.t.

$$\alpha u(x) + \beta = \int \max\{v(x), k\} F(dk) \quad \forall x \in X.$$

2 \Rightarrow 1 *Positive*: effective risk attitude u is a lower-bound on “true” attitude v

1 \Rightarrow 2 *Negative*: nothing more can be learned about v from u .

Theorem 1

Proposition 1: o.o.s act as increasing, convex transformations of v and reduce risk aversion.

Theorem 1: an equivalence. Every admissible “flattening” of v is exactly “add an outside option.”

What do choices reveal about o.o.?

Proposition 2. For u and xCDF F , the following are equivalent

1. $\int_{(0,+\infty)} kF(dk) < +\infty$ and if $\int_{(-\infty,0]} kF(dk) > -\infty$ then u is bounded below.
2. $\exists \alpha, \beta \in \mathbb{R}, \alpha > 0$, and $v: X \rightarrow \mathbb{R}$ s.t.

$$\alpha u(x) + \beta = \phi(u(x)) = \int \max\{v(x), k\} F(dk) \quad \forall x \in X.$$

V. negative: observation of 'tude u tells us nothing about F except that $\int_{(0,+\infty)} kF(dk) < +\infty$ and, if u is unbounded below, that $\int_{(-\infty,0]} kF(dk) = -\infty$.

Proposition 2

- ▶ **Non-identification of the fallback.** From observed effective attitude u alone, the outside-option distribution F is *not* identified beyond tail-integrability facts. *Many F can rationalize the same u .*
- ▶ **Re-labeling invariance.** For any F satisfying the tail conditions, one can choose $\alpha > 0, \beta \in \mathbb{R}$ and define v so that

$$\alpha u(x) + \beta = \int \max\{v(x), k\} F(dk),$$

i.e., different F are observationally equivalent once v is re-parameterized.

- ▶ **What is identified.** Theorem 1 implies only that the *true* attitude v is (weakly) more risk-averse than u ; there is no sharper information about F without extra structure or variation.

Comparative Statics Preliminary

Given $K \subseteq \mathbb{R}$ x Cdfs F and \hat{F} , say that \hat{F} is better than F in the reverse hazard rate order on K if and only if

$$F(\ell)\hat{F}(k) \leq F(k)\hat{F}(\ell) \quad \forall k, l \in K, k < l$$

If K is an open interval and F, \hat{F} are C^1 w/ $F, \hat{F} > 0$ on $(\inf K, +\infty)$, equivalent to $\hat{F}'/\hat{F} \geq F'/F$ on K .

Likelihood-ratio dominance \Rightarrow RHR dominance \Rightarrow FOSD.

Theorem 2. Fix outside-option representations

$$\alpha u(x) + \beta = \int \max\{v(x), k\} F(dk), \quad \hat{\alpha} \hat{u}(x) + \hat{\beta} = \int \max\{\hat{v}(x), k\} \hat{F}(dk),$$

with F concentrated on $v(X) \cup \{-\infty\}$ and \hat{F} on $\hat{v}(X) \cup \{-\infty\}$. Then:

- ▶ Holding v fixed: \hat{u} is less RA than $u \Leftrightarrow \hat{F} \succeq_{\text{RHR}} F$ on $v(X) \setminus \{\sup v(X)\}$.
- ▶ Holding the *physical* outside-option value distribution fixed (i.e. $F \circ v = \hat{F} \circ \hat{v}$): \hat{u} is less RA than $u \Leftrightarrow \hat{v}$ is less RA than v .

Smooth Special Case

Assume $X \subset \mathbb{R}$ is open and convex, $v, \hat{v} \in C^2$ with $v', \hat{v}' > 0$, and $F, \hat{F} \in C^1$. Let $G := F \circ v$ and $\hat{G} := \hat{F} \circ \hat{v}$. Then

$$\frac{u''}{u'} = \frac{v''}{v'} + \frac{G'}{G}, \quad \& \quad \frac{\hat{u}''}{\hat{u}'} = \frac{\hat{v}''}{\hat{v}'} + \frac{\hat{G}'}{\hat{G}}.$$

- ▶ If $v = \hat{v}$: \hat{u} is less RA than $u \Leftrightarrow \hat{G}'/\hat{G} \geq G'/G$ (equivalently, $\hat{F} \succeq_{\text{RHR}} F$ on $v(X) \setminus \{\sup v(X)\}$).
- ▶ If $G = \hat{G}$: \hat{u} is less RA than $u \Leftrightarrow \hat{v}$ is less RA than v .

Recall *Likelihood-ratio dominance* \Rightarrow *RHR dominance* \Rightarrow *FOSD*.

What's special (if anything) about adding an o.o.?

Adding an o.o. \Rightarrow transforms problem by replacing each $x \in X$ w/ an x -contingent lottery.

There are other transformations...

E.g., adding background risk?

Are there other transformations that decrease risk aversion?

What's special (if anything) about adding an o.o.?

Formally, replace each x w/ draw from measure μ_x on X .

“True” v versus effective via $x \mapsto \int v d\mu_x$

Which families $(G_x)_{x \in X}$ of CDFs concentrated on X have the property that \forall strictly increasing $v: X \rightarrow \mathbb{R}$, $x \mapsto \int v dG_x$ is less risk-averse than v ?

What's special about o.o.s?

Theorem 3. Let $X \subset \mathbb{R}$ be Borel w/ $\sup X \notin X$, and let $\{G_x\}_{x \in X}$ be CDFs on X . The following are equivalent:

1. \forall bounded, strictly increasing $v: X \rightarrow \mathbb{R}$, $x \mapsto \int v dG_x$ is less RA than v .
2. There exists an xCDF G concentrated on $X \cup \{-\infty\}$ with $G > 0$ on $(\inf X, \infty)$ s.t. for every bounded, strictly increasing v , there exist $\alpha > 0, \beta \in \mathbb{R}$ s.t.

$$\alpha \int v dG_x + \beta = \int \max\{v(x), v(y)\} G(dy) \quad \forall x \in X,$$

with the convention $v(-\infty) := -\infty$.

Theorem 3, in words

- ▶ Take a rule that replaces each alternative x by a lottery G_x on X and evaluates x via $\int v(y) G_x(dy)$, where $v: X \rightarrow \mathbb{R}$ is any bounded, strictly inc. risk attitude.
- ▶ Suppose this rule is always less risk-averse than v .
- ▶ This is true if and only if the rule is an o.o. in disguise: rule is “take the better of inside alternative and an x -independent outside draw.”
- ▶ O.o.s: not merely one way to reduce risk aversion; the only way to do so uniformly across all possible true (increasing) preferences.

Intuition

- ▶ The uniform requirement “less RA for *all* increasing v ” forbids using the specific curvature of v ; the transform must act only via the *value* $v = v(x)$.
- ▶ O.o.s do exactly that: $\phi(v) = \mathbb{E}[\max\{v, K\}]$ has slope $\phi'(v) = \mathbb{P}[K \leq v]$, an xCDF depending only on v .
- ▶ In contrast, background risk or ad hoc perturbations interact with v''/v' ; they can raise or lower RA depending on v , so they cannot pass the “for all v ” test.

(2 \implies 1) By Theorem 1.

One use

Weinstein ('16), Battigalli et al. ('16): decreased risk aversion expands rationalizable (undominated) set.

Curello et al. ('25): a converse. Universal expansions \iff greater risk propensity!

Combine: model is lower bound on what can be rationalized!

Actions not purely dominated \equiv actions rationalized by some (convex) transformation of utility. This is the set of actions that can be rationalized by o.o.!

Another use (time permitting)

Pease & Whitmeyer ('23) "Playing it Safe: Actions Attractive to the Risk Averse"

What makes one action (act) "riskier" than another?

SEU world: state-space Θ ; DM w/ a set of actions $A \subseteq \mathbb{R}^\Theta$, each continuous (and none dominated by another); utility $u: \mathbb{R} \rightarrow \mathbb{R}$.

a is *safer* than b , $a \succeq_S b$, if \forall concave & strictly-inc. ϕ , strictly inc. u , and $\mu \in \Delta(\Theta)$,

$$\mathbb{E}_\mu u(a_\theta) \geq \mathbb{E}_\mu u(b_\theta) \implies \mathbb{E}_\mu \phi \circ u(a_\theta) \geq \mathbb{E}_\mu \phi \circ u(b_\theta).$$

PW main theorem

Define

$$\mathcal{A} := \{\theta \in \Theta : a_\theta > b_\theta\} \quad \text{and} \quad \mathcal{B} := \{\theta \in \Theta : a_\theta < b_\theta\}.$$

PW Theorem. The following are equivalent:

1. $a \succeq_S b$.
2. For any pair $(\theta, \theta') \in \mathcal{A} \times \mathcal{B}$, $b_{\theta'} \geq a_\theta$ and $a_{\theta'} \geq b_\theta$.
3. For all $\mu \in \Delta$, $F_a^\mu \succeq_{sc} F_b^\mu$.
4. For all $\mu \in \Delta$, if $\mathbb{E}_\mu a_\theta \geq \mathbb{E}_\mu b_\theta$, then there exists an RV \tilde{X} such that X_a^μ FOSD \tilde{X} and \tilde{X} SOSD X_b^μ . Moreover, if $\mathbb{E}_\mu a_\theta = \mathbb{E}_\mu b_\theta$, then X_a^μ SOSD X_b^μ .

Ex. Security demand

State $\theta \in [0, 1]$, security pays $S(\theta)$ to an investor. E.g.,

1. Equity: the investor receives a constant portion, $\eta \in (0, 1)$, of the cash flow, or $S(\theta) = \eta\theta$,
2. Debt: the investor is owed a debt $d \in (0, 1)$ and collects as much of it as possible, or $S(\theta) = \min\{\theta, d\}$; and
3. Call Option: the investor gets a call option with a strike price of $\rho \in (0, 1)$, or $S(\theta) = \max\{\theta - \rho, 0\}$.

General rank: $S_a \succeq_S S_b$ if and only if $S_b \succeq_{sc} S_a$.

⇒ Debt safer than all, call option least safe!

O.o. ⇒ demand shifts predictably!

Break?

Proofs.

How does an o.o. shape risk attitude?

Proposition 1. For $X \neq \emptyset$, $u, v: X \rightarrow \mathbb{R}$ and xCDF F , the following are equivalent.

1. $\exists \alpha, \beta \in \mathbb{R}, \alpha > 0$, s.t.

$$\alpha u(x) + \beta = \int \max\{v(x), k\} F(dk) \quad \forall x \in X.$$

2. $\int_0^\infty kF(dk) < \infty$ & \exists a.c. $\phi: \text{co}(v(X)) \rightarrow \mathbb{R}$ s.t. $u = \phi \circ v$, and $\phi' \propto F$ a.e.

When i. everything is money, ii. open & convex $X \subseteq \mathbb{R}$, iii. u, v are C^2 w/ $u', v' > 0$, and iv. $F = G \circ v^{-1}$ for xCDF G

$$2 \quad \iff \quad \frac{u''}{u'} = \frac{v''}{v'} + \overbrace{\frac{G'}{G}}^{\text{reverse hazard rate!}}$$

Proposition 1 sketch (outside option \Rightarrow convex transform)

From $\alpha u(x) + \beta = \int \max\{v(x), k\} F(dk)$ WTS $u = \phi \circ v$ with $\phi' = \lambda F$ (a.e.).

Define $\phi(v) := \int \max\{v, k\} F(dk)$. Differentiate under the integral \Rightarrow

$$\phi'(v) = \int \partial_v \max\{v, k\} F(dk) = \int \mathbf{1}_{\{k \leq v\}} F(dk) = F(v) \quad (\text{a.e.}).$$

Proposition 1 sketch (convex transform \Rightarrow outside option)

Assume $u = \phi \circ v$ with ϕ increasing, convex, absolutely continuous, and $\phi' = \lambda F$ a.e. for some $\lambda > 0$ (plus $\int_{(0,\infty)} k F(dk) < +\infty$).

Let $H(v) := \int \max\{v, k\} F(dk)$. Then as above $H'(v) = F(v)$ (a.e.) and

$$H(v) = H(0) + \int_0^v F(t) dt, \quad \text{and} \quad \phi(v) = \phi(0) + \lambda \int_0^v F(t) dt.$$

Thus, $\phi(v) = \lambda H(v) + (\phi(0) - \lambda H(0))$. Setting $\alpha := \frac{1}{\lambda}$ and $\beta := H(0) - \frac{1}{\lambda} \phi(0)$ yields

$$\alpha u(x) + \beta = H(v(x)) = \int \max\{v(x), k\} F(dk).$$

What do choices reveal about “true” risk attitude?

Theorem 1. Suppose $u, v: X \rightarrow \mathbb{R}$ are bounded above and u is Lipschitz with respect to v . The following are equivalent:

1. u is less risk-averse than v .
2. There exist $\alpha > 0, \beta \in \mathbb{R}$, and an xCDF F (with $F > 0$ on $(\inf v(X), \infty)$) s.t.

$$\alpha u(x) + \beta = \int \max\{v(x), k\} F(dk) \quad \forall x \in X.$$

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Theorem 1

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Theorem 1: an equivalence. Every admissible “flattening” of v is exactly “add an outside option.”

Theorem 1 sketch

(1 \implies 2)

1. Start from what “less RA” means. By Pratt/Yaari, there is an increasing, convex ψ with $u = \psi \circ v$.
2. Look at slopes. Convexity \implies the right derivative ψ'_+ exists and is nondecreasing; Lipschitz of u in $v \implies \psi'_+$ is bounded. Normalize a scale $\alpha > 0$ and define

$$F(v) := \alpha \psi'_+(v).$$

Then F is nondecreasing, right-continuous, $0 \leq F \leq 1$: an xCDF!

3. Integrate the slope. Recover ψ by

$$\psi(v) = \psi(0) + \frac{1}{\alpha} \int_0^v F(t) dt.$$

Theorem 1 sketch (II)

4. Identify the “outside-option.” For any xCDF F , set $H(v) := \int \max\{v, k\} F(dk)$. Then $H'(v) = F(v)$ (a.e.) and, anchoring at 0,

$$H(v) = H(0) + \int_0^v F(t) dt.$$

Thus $\psi(v) = \frac{1}{\alpha} H(v) + \psi(0) - \frac{1}{\alpha} H(0)$.

5. Compose back and tidy constants. Since $u = \psi \circ v$, with $v = v(x)$,

$$\alpha u(x) + \beta = H(v(x)) = \int \max\{v(x), k\} F(dk),$$

where $\beta := H(0) - \alpha\psi(0)$.

(1 \iff 2) **Easy direction (already in Prop. 1).** Given an o.o. with $F > 0$ on $(\inf v(X), \infty)$, Prop. 1 gives $u = \psi \circ v$ with ψ increasing and convex, and $F > 0$ ensures ψ is strictly increasing on $v(X)$; hence, u is less RA than v .

Theorem 2. Fix outside-option representations

$$\alpha u(x) + \beta = \int \max\{v(x), k\} F(dk), \quad \hat{\alpha} \hat{u}(x) + \hat{\beta} = \int \max\{\hat{v}(x), k\} \hat{F}(dk),$$

with F concentrated on $v(X) \cup \{-\infty\}$ and \hat{F} on $\hat{v}(X) \cup \{-\infty\}$. Then:

- ▶ Holding v fixed: \hat{u} is less RA than $u \Leftrightarrow \hat{F} \succeq_{\text{RHR}} F$ on $v(X) \setminus \{\sup v(X)\}$.
- ▶ Holding the *physical* outside-option value distribution fixed (i.e. $F \circ v = \hat{F} \circ \hat{v}$): \hat{u} is less RA than $u \Leftrightarrow \hat{v}$ is less RA than v .

Smooth Special Case

Assume $X \subset \mathbb{R}$ is open and convex, $v, \hat{v} \in C^2$ with $v', \hat{v}' > 0$, and $F, \hat{F} \in C^1$. Let $G := F \circ v$ and $\hat{G} := \hat{F} \circ \hat{v}$. Then

$$\frac{u''}{u'} = \frac{v''}{v'} + \frac{G'}{G}, \quad \& \quad \frac{\hat{u}''}{\hat{u}'} = \frac{\hat{v}''}{\hat{v}'} + \frac{\hat{G}'}{\hat{G}}.$$

- ▶ If $v = \hat{v}$: \hat{u} is less RA than $u \Leftrightarrow \hat{G}'/\hat{G} \geq G'/G$ (equivalently, $\hat{F} \succeq_{\text{RHR}} F$ on $v(X) \setminus \{\sup v(X)\}$).
- ▶ If $G = \hat{G}$: \hat{u} is less RA than $u \Leftrightarrow \hat{v}$ is less RA than v .

Recall *Likelihood-ratio dominance* \Rightarrow *RHR dominance* \Rightarrow *FOSD*.

What's special about o.o.s?

Theorem 3. Let $X \subset \mathbb{R}$ be Borel w/ $\sup X \notin X$, and let $\{G_x\}_{x \in X}$ be CDFs on X . The following are equivalent:

1. \forall bounded, strictly increasing $v: X \rightarrow \mathbb{R}$, $x \mapsto \int v dG_x$ is less RA than v .
2. There exists an xCDF G concentrated on $X \cup \{-\infty\}$ with $G > 0$ on $(\inf X, \infty)$ s.t. for every bounded, strictly increasing v , there exist $\alpha > 0, \beta \in \mathbb{R}$ s.t.

$$\alpha \int v dG_x + \beta = \int \max\{v(x), v(y)\} G(dy) \quad \forall x \in X,$$

with the convention $v(-\infty) := -\infty$.

(2 \implies 1) By Theorem 1.

Theorem 3: (1 implies 2)

1. Prove a stronger shape first

Let $X = (a, b)$ and assume for every bounded, C^2 , and strictly increasing v ,

$$u_v(x) := \int_X v(y) G_x(dy)$$

is less risk-averse than v .

Observe: enough to show $G_x = \lambda G \mathbf{1}_{[x, \infty)} + (1 - \lambda)H \forall x \in X$, where $\lambda \in (0, 1]$, x -CDF G and CDF H .

Then,

$$\underbrace{\frac{1}{\lambda}}_{\alpha} \int v dG_x + \overbrace{\left(1 - \frac{1}{\lambda}\right)}^{\beta} \int v dH = \int v d[G \mathbf{1}_{[x, \infty)}] = \int \max\{v(x), v(y)\} G(dy) \quad \forall x \in X.$$

So, why must the family (G_x) have this shape?

Theorem 3: proof idea (1 implies 2) II

2. Locality

As u_v is less risk-averse than v , transformation is order preserving: for $x < y$,

$$\begin{aligned} v(x) < v(y) &\implies \int v dG_x < \int v dG_y \quad \forall \text{ bounded, strictly increasing, } C^2 v \\ &\implies G_y \succeq_{\text{FOSD}} G_x. \end{aligned}$$

Claim 1: $G_x = G_y$ on $\mathbb{R} \setminus [x, y)$.

- ▶ If G_x and G_y differed outside $[x, y)$, choose a test utility that is almost flat on $[x, y]$ but steep where the illegal difference sits.
- ▶ Breaks Pratt's secant-slope inequality.

Theorem 3: proof idea (1 implies 2) III

3. One common wedge

Define, for $t \in X$,

$$L(t) := \inf_z G_z(t), \quad R(t) := \sup_z G_z(t), \quad \text{and} \quad D(t) := R(t) - L(t).$$

Claim $1 + x \mapsto G_x(t)$ decreasing \Rightarrow

$$G_x(t) = \begin{cases} L(t), & \text{if } t < x, \\ R(t), & \text{if } t \geq x. \end{cases} \quad \Rightarrow \quad G_x = L + \mathbf{1}_{[x, \infty)} D,$$

so

$$u_v(x) = \int_X v(y) G_x(dy) = \int v dL + D(x)v(x) + \int_{(x, \infty)} v(y) D(dy),$$

so $u'_v(x) = v'(x)D(x)$, so

$$\frac{u''_v(x)}{u'_v(x)} = \frac{v''(x)D(x) + v'(x)D'(x)}{v'(x)D(x)} = \frac{v''(x)}{v'(x)} + \frac{D'(x)}{D(x)}.$$

The same D'/D to every v !

Theorem 3: proof idea (1 implies 2) IV

4. Increasing wedge = outside option

And

$$\frac{u_v''(x)}{u_v'(x)} \geq \frac{v''(x)}{v'(x)} \quad \Longrightarrow \quad \frac{D'(x)}{D(x)} \geq 0 \quad \Longrightarrow \quad D'(x) \geq 0$$

Finally, normalize

$$\lambda := \sup D, \quad G := D/\lambda, \quad \text{and} \quad H := L/(1-\lambda) \quad (\lambda < 1).$$

Thus,

$$G_x = \lambda G \mathbf{1}_{[x, \infty)} + (1-\lambda)H.$$

and so

$$\frac{1}{\lambda} \int v dG_x + \left(1 - \frac{1}{\lambda}\right) \int v dH = \int \max\{v(x), v(y)\} G(dy) \quad \forall x$$

\Longrightarrow fixed outside option.

Thank you!