Comparing Experiments in Discounted Problems

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Dynamic decision problems

States: Θ, w/ generic element θ.

Time: $t = 1, \ldots, T$ ($T \leq \infty$).

Dynamic information structures:

$$f : \Theta \to \Delta(X_1 \times \cdots \times X_T), \qquad \text{vs.} \qquad g : \Theta \to \Delta(Y_1 \times \cdots \times Y_T).$$

Dynamic decision problem: @ each t DM chooses $a_t \in A_t$ after observing past and current signals... *but not future ones.*

Utility: $u(a_1, \ldots, a_T, \theta)$.

All sets finite, except (possibly) # of pds.

Discounted problems

Additively-separable (AS) problem:

$$u(a_1, \ldots, a_T, \theta) = \sum_{t=1}^{T} u_t(a_t, \theta),$$

for some cllxn. $(u_t : A_t \times \Theta \to \mathbb{R})_t$

Discounted problem: Common $A = A_t \; \forall t$, common $u : A \times \Theta \to \mathbb{R}$, &

$$u(a_1, \ldots, a_T, \theta) = \sum_{t=1}^{T} \delta_t u(a_t, \theta), \quad \text{for some } (\delta_t)_t \in \Delta(\{1, \ldots, T\}).$$

Misc. subclasses: all, decreasing, decreasing & convex, exponential, fixed.

Comparisons of info structures

Static problems (dynamic w/ $T = 1$). Blackwell ('51, '53).

Dynamic problems. Greenshtein ('96).

Discounted problems. This talk.

Questions

1. Comparisons of info structures for discounted problems?

2. De Jarnette, Dillenberger, Gottlieb, Ortoleva (′20): attitudes toward timing risk. Here: timing risk over info arrival?

Roadmap

1. Brief review of Blackwell (’51, ’53) & Greenshtein (’96).
2. Some examples.
3. Discounted problems: fixed discount factor.
4. Discounted problems: sets of discount factors + AS problems.
5. Risk preferences over info arrival.
6. Preview: more questions.

Review

Review of Blackwell ('51, '53)

Single pd. dynamic problem: A finite set Θ, w/ generic element θ.

Information: Two statistical experiments $f\colon \Theta \to \Delta(X)$ and $g\colon \Theta \to \Delta(Y)$ to be compared.

Decision problem: (A, u), where A is the set of decisions and $u : A \times \Theta \to \mathbb{R}$ the utility/loss function.

Strategies: $\sigma\colon X \to \Delta(A)$ and $\tau\colon\ Y \to \Delta(A)$.

Distributions: $\mathbb{P}_{\theta,f,\sigma}$ and $\mathbb{P}_{\theta,g,\tau}$ are the induced distributions over signals and actions (given state θ).

Review of Blackwell ('51, '53)

Definition

The experiment f is *more valuable* than the experiment g if for all decision problems (A, u), for any strategy τ, there exists a strategy σ such that

$$\mathbb{E}_{\theta,g,\tau}[u(\mathbf{a},\theta)] \leq \mathbb{E}_{\theta,f,\sigma}[u(\mathbf{a},\theta)] \quad \forall\, \theta$$

Key: The information is fixed, but *all* decision problems are considered.

Remark: No prior is considered, hence "for all θ."

Given a prior $\mu_0 \in \Delta(\Theta)$, f induces (*ex-ante*) distribution over posteriors $F \in \Delta(\Delta(\Theta))$ w/ $\mathbb{E}_F(\mu) = \mu_0$.

Review of Blackwell ('51, '53)

Marginal dist. over actions: $(\mathrm{marg}_A \mathbb{P}_{\theta,g,\tau})(a) := \sum_{y \in Y} \mathbb{P}_{\theta,g,\tau}(y,a)$.

Theorem

T.f.a.e:

1. *The experiment f is more valuable than the experiment g.*
2. *For all decision problems (A,u), for all τ, there exists σ such that*

$$\mathrm{marg}_A \mathbb{P}_{\theta,g,\tau} = \mathrm{marg}_A \mathbb{P}_{\theta,f,\sigma}, \quad \forall\ \theta.$$

3. *Sufficiency: There exists a garbling $\gamma: X \to \Delta(Y)$ such that*

$$g(y|\theta) = \sum_x \gamma(y|x) f(x|\theta), \quad \forall\ y, \forall\ \theta$$

4. *For all $\mu_0 \in \Delta(\Theta)$, $F \geq_{cx} G$.*

Back to dynamics

Recall $f\colon \Theta \to \Delta(X_1 \times \cdots \times X_T)$ & utility $u(a_1, \ldots, a_T, \theta)$.
Write

$$X^t = X_1 \times \cdots \times X_t, \qquad \& \quad A^t = A_1 \times \cdots \times A_t.$$

Strategy: a stochastic map

$$\alpha\colon X^T \to \Delta(A^T)$$

that is Adapted: for each t, the marginal

$$\alpha^t(a_1, \ldots, a_t \mid x_1, \ldots, x_T) = \sum_{a_{t+1}, \ldots, a_T} \alpha(a_1, \ldots, a_T \mid x_1, \ldots, x_T)$$

depends only on $(x_1, \ldots, x_t)$.

$$\Theta \xrightarrow{f} X^T \xrightarrow{\alpha} A^T.$$

Key observation: strategies and garblings are the same type of object.

Review of Greenshtein ('96)

Definition

The dynamic information structure f is *more informative* than g if for all dynamic decision problems and every adapted strategy τ for g, there exists an adapted strategy σ for f such that

$$\mathbb{E}_{\theta,g,\tau}[u(\mathbf{a}_1,\ldots,\mathbf{a}_T,\theta)] \leq \mathbb{E}_{\theta,f,\sigma}[u(\mathbf{a}_1,\ldots,\mathbf{a}_T,\theta)] \quad \forall\, \theta.$$

Key: mirrors Blackwell, only now the actions and signals arrive over time.

Implementable distributions over action histories:

$$\Lambda_f(A_1,\ldots,A_T) = \{\alpha \circ f : \ \alpha \text{ adapted}\}.$$

Expected utility depends only on the element of $\Lambda_f(A_1,\ldots,A_T)$ that is induced.

Review of Greenshtein ('96). Also De Oliveira ('17)

Theorem

T.f.a.e:

1. f *is more informative than* g *in every dynamic decision problem.*
2. *For every action space* $(A_t)_t$,

$$\Lambda_f(A_1,\ldots,A_T) \supseteq \Lambda_g(A_1,\ldots,A_T).$$

3. *There exists an adapted garbling*

$$\gamma\colon X^T \to \Delta(Y^T)$$

   s.t. $g = \gamma \circ f$.

4. *Sequences of distributions over posteriors?*

Blackwell's siren call

For each t, view dynamic experiment f as static experiment $f^t \to \Delta(X^t)$.

Why not $f^t \geq_B g^t$ for all t?

Equivalently, $F^t \geq_{cx} G^t$ for all $\mu_0 \in \Delta(\Theta)$, for all t.

Examples

Example 1: a stopping problem

Two states and two actions:

$$\theta \in \{0,1\}, \qquad a \in \{0,1\}, \qquad \& \qquad \mu_0 = \mathbb{P}(\theta = 1).$$

Two conditionally-independent binary signals: Weak and Strong:

	$\mathbb{P}(W = 1 \mid \theta)$	$\mathbb{P}(S = 1 \mid \theta)$
$\theta = 0$	$1/3$	$1/4$
$\theta = 1$	$2/3$	$3/4$

Two sequential experiments:

$$f = (W, S), \qquad \& \qquad g = (S, W).$$

The first signal is “free.” Observing the second signal costs $c = 1/100$.

Example 1: utilities

Payoffs:

	$\theta = 0$	$\theta = 1$
$a = 0$	1	0.9
$a = 1$	0	1

Plus: if the second signal is bought, subtract $c = 1/100$.

$$v(q) = \max\left\{1 - \frac{q}{10}, q\right\}.$$

Static cutoff:

$$a = 1 \iff q \geq \frac{10}{11}.$$

Example 1: decisions

Set $\mu_0 = \mathbb{P}(\theta = 1) = \frac{4}{5}$.

Supports are

$$f: \quad \frac{4}{5} \to \left\{ \frac{2}{3}, \overset{w.p.\ 12/20}{\frac{8}{9}} \right\} \to \left\{ \frac{2}{5}, \frac{8}{11}, \frac{6}{7}, \frac{24}{25} \right\},$$

$$g: \quad \frac{4}{5} \to \left\{ \frac{4}{7}, \underset{w.p.\ 13/20}{\frac{12}{13}} \right\} \to \left\{ \frac{2}{5}, \frac{8}{11}, \frac{6}{7}, \frac{24}{25} \right\}.$$

Example 1.5: discounting

Previous decision problem: not a discounted problem.

Not AS.

Take arbitrary AS problem; for $t = 1, 2$ and $p \in [0, 1]$ write $V_t(p) := \max_{a_t} \mathbb{E}_p u_t(a_t, \theta)$.

$$\mathbb{E}_{G_1} V_1 \geq \mathbb{E}_{F_1} V_1 \;\&\; \mathbb{E}_{G_2} V_2 \geq \mathbb{E}_{F_2} V_2 \quad \Longrightarrow \quad g \text{ better for AS problems!}$$

$\Longrightarrow$ g superior to f for discounted problems (all varieties).

Example 2: info arrival

Fix an *informative* Blackwell experiment $\pi\colon \Theta \to \Delta(S)$.

R.V. $\mathcal{T}$: realized t is when the DM gets to see draw from π.

Discounted problems:

$$u(a_1, \ldots, a_T, \theta) = \sum_{t=1}^{T} \delta_t u(a_t, \theta), \quad \text{for some } (\delta_t)_t \in \Delta(\{1, \ldots, T\}).$$

Risk attitudes toward arrival? E.g., $T = 3$: prefer @ $t = 2$ for sure vs. 50/50 1 or 3.

Example 2: consumption streams benchmark

De Jarnette, Dillenberger, Gottlieb, Ortoleva ('20): prize x @ random time w/ $u(x) > u(0) = 0$.

Compare $\delta_t u(x)$ w/ $\mathbb{E}\delta_{\mathcal{T}} u(x)$.

Shape of δ_t v. important: exponential $\delta_t = \beta^t$ is convex $\Longrightarrow$ risk-seeking over time lotteries.

What about over information?

Example 2: 3 periods

Suppose $T = 3$ and $\delta_1 = 1/2$ $\delta_2 = 2/5$ & $\delta_3 = 1/10$.

Re. De Jarnette et. al.: set $u(x) = 1$.

$$\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{10} = \frac{3}{10} < \frac{4}{10} = 1 \times \frac{2}{5} \quad \Longrightarrow \quad \text{Risk averse!}$$

Now decision problem: π informative, VOI positive for some us strictly so.

Normalize no info value to 0, value to $\alpha \geq 0$.

$$\left[\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{10}\right]\alpha \ \underbrace{+\frac{1}{2}\left[\frac{2}{5} + \frac{1}{10}\right]\alpha}_{\text{WHY?}} \quad \text{vs.} \quad \frac{2}{5}\alpha \ \underbrace{+\frac{1}{10}\alpha}_{\text{WHY?}}$$

$$\frac{11}{20}\alpha > \frac{10}{20}\alpha \quad \Longrightarrow \quad \text{Risk seeking!}$$

The Main Result

The discounted model – reminder

States: A set Θ, with generic element θ.

Horizon: $T \leq +\infty$ periods, w/ generic element t.

Information: Two statistical experiments $f := (f_t)_t$ and $g := (g_t)_t$ w/

$$f_t : \Theta \times X^{t-1} \to \Delta(X_t), \qquad \& \qquad g_t : \Theta \times Y^{t-1} \to \Delta(Y_t)$$

AS Decision problems: $(A_t, u_t)_t$, w/ A_t is the set of decisions @ t, $u_t : A_t \times \Theta \to \mathbb{R}$ the utility @ t.

Notation. $X^t := X_1 \times \cdots \times X_t$, etc.

The discounted model – reminder

Discounted problem: $(A_t, u_t) = (A, u)$ for all t, & DM evaluates $(a_t)_t$ as

$$\sum_t \delta_t u(a_t, \theta), \qquad \text{for some } (\delta_t)_t \in \Delta(\{1, \ldots, T\}).$$

AS problem: DM evaluates $(a_t)_t$ as

$$\sum_t u_t(a_t, \theta).$$

Strategies:

$$\sigma_t : X^t \times A^{t-1} \to \Delta(A_t) \qquad \& \quad \tau_t : Y^t \times A^{t-1} \to \Delta(A_t).$$

Distributions: $\mathbb{P}_{\theta,f,\sigma}$ and $\mathbb{P}_{\theta,g,\tau}$ are the induced distributions over signals and actions (given θ).

Discounted problems – comparing experiments

Definition

Let δ be the discount factor. The statistical experiment f is *more valuable* than g in discounted problems, if for all decision problems (A, u), for any strategy τ, there exists strategy σ such

$$\mathbb{E}_{\theta,g,\tau}\left[\sum_t \delta_t u(\mathbf{a}_t, \theta)\right] \leq \mathbb{E}_{\theta,f,\sigma}\left[\sum_t \delta_t u(\mathbf{a}_t, \theta)\right], \quad \forall\, \theta$$

Key: Discount factor is fixed in the basic model.

Aim: Derive the analog of Blackwell's theorem.

Discounted problems – a lemma

Time-weighted marginal dist. over actions:
$(\sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,g,\tau})(a) := \sum_t \sum_{y^t \in Y^t} \delta_t \mathbb{P}_{\theta,g,\tau}(y^t, a)$
Note:

$$\mathbb{E}_{\theta,f,\sigma}\left[\sum_t \delta_t u(\mathbf{a}_t, \theta)\right] = \sum_a u(a,\theta) \underbrace{\left[\sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,f,\sigma}(a)\right]}_{\text{discounted prob. of "}a\text{"}}.$$

Hence, as in static problems ((1) $\iff$ (2) in Blackwell):

Lemma. f is more valuable than g in all discounted problems if, and only if, for all τ, there exists σ, such that

$$\sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,g,\tau} = \sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,f,\sigma}, \quad \forall\, \theta.$$

Discounted problems – Garblings

Now want analog between (2) $\Longleftrightarrow$ (3) in Blackwell, *viz.*, via garblings.

From the lemma, restrict attention to ensuring that:

$$\sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,g,\tau} = \sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,f,\sigma},$$

i.e., matching the discounted probabilities of decisions.

Discounted problems – the baseline

We have:

$$\sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,f,\sigma}(a) = \sum_t \sum_{x^t \in X^t} \delta_t \mathbb{P}_{\theta,f,\sigma}(x^t) \mathbb{P}_{\theta,f,\sigma}(a|x^t).$$

Moreover, $\mathbb{P}_{\theta,f,\sigma}(x^t)$ does not depend on σ and is equal to:

$$f^t(x^t|\theta) := f_1(x_1|\theta) \times \cdots \times f_t(x_t|x^{t-1}, \theta).$$

So, we can rewrite the above in the form:

$$\sum_t \sum_{x^t \in X^t} \underbrace{\delta_t f^t(x^t|\theta)}_{\text{info}} \underbrace{\overline{\sigma}(a|t, x^t)}_{\text{strategy}}.$$

Punchline: Back to a static problem, with the reinterpretation of a signal as being "(t, x^t)" with prob. $\delta_t f^t(x^t|\theta)$, when state is θ.

Similarly,

$$\sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,g,\tau}(a) = \sum_t \sum_{y^t \in Y^t} \delta_t g^t(y^t|\theta)\overline{\tau}(a|t,y^t).$$

From (2) $\iff$ (3) in Blackwell, need the existence of a garbling γ satisfying

$$\delta_t g^t(y^t|\theta) = \sum_{t',x^{t'}} \delta_{t'} f^{t'}(x^{t'}|\theta)\gamma(\underbrace{t,y^t}_{\text{“simulations”}}|t',x^{t'}),$$

for all θ, for all (t,y^t).

Call this condition δ-sufficiency.

Intuition. It does not matter when “a” is chosen, as long as the averages match.

Discounted problems – the baseline

Example: Info today vs. info tomorrow.

Two periods, equally discounted $(\delta_1, \delta_2) = (1/2, 1/2)$.

- f: no info in period 1, but full info in period 2.
- g: some info in period 1, but no further info in period 2.

For a numerical example,

$\theta \backslash x$	x_0	x_1
θ_0	1	0
θ_1	0	1

Table: f

$\theta \backslash y$	y_0	y_1
θ_0	7/12	5/12
θ_1	5/12	7/12

Table: g

Clearly, f is more informative than g in static problems, but is it "sufficiently more informative"?

Discounted problems – the baseline

δ-sufficiency amounts to comparing a modified version of f with g, where

$\theta \backslash x$	x_0	x_1	$\emptyset$
θ_0	$1/2$	0	$1/2$
θ_1	0	$1/2$	$1/2$

Table: the modified f

Modified f: with prob. $1/2$, no info; with prob. $1/2$, full info.

Answer: The "modified f" is sufficient for g, hence f is δ-sufficient for g.

Alternative approach

Given $\mu_0 \in \Delta(\Theta)$, sequences of distributions over posteriors:

$$(F_t)_t \qquad \text{vs.} \qquad (G_t)_t.$$

Define

$$F^\delta := \sum_{t=1}^{T} \delta_t F_t \quad \& \quad G^\delta := \sum_{t=1}^{T} \delta_t G_t.$$

Proposition

f m.v. than g in discounted problems w/ factor δ if and only if $F^\delta \geq_{cx} G^\delta \; \forall \; \mu_0$.

Alternative approach

Proposition

f m.v. than g in discounted problems w/ factor δ if and only if $F^\delta \geq_{cx} G^\delta \ \forall \ \mu_0$.

proof.

$$\sum_{t=1}^{T} \delta_t \mathbb{E}_{F_t} V(\mu) \geq \sum_{t=1}^{T} \delta_t \mathbb{E}_{G_t} V(\mu) \ \forall \text{ convex } V \iff \mathbb{E}_{\sum_{t=1}^{T} \delta_t F_t} V(\mu) \geq \mathbb{E}_{\sum_{t=1}^{T} \delta_t G_t} V(\mu)$$

$$\iff \mathbb{E}_{F^\delta} V(\mu) \geq \mathbb{E}_{G^\delta} V(\mu).$$

Discounted problems – the basics

Theorem

T.f.a.e:

1. *f m.v. than g in discounted problems w/ factor δ.*
2. *For all decision problems (A, u), for all strategies τ, there exists a strategy σ s.t.*

$$\sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,g,\tau} = \sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,f,\sigma}, \quad \forall\, \theta.$$

3. *δ-sufficiency: There exist garbling γ s.t.*

$$\delta_t g^t(y^t|\theta) = \sum_{t'} \sum_{x^{t'} \in X^{t'}} \delta_{t'} f^{t'}(x^{t'}|\theta)\gamma(t, y^t \mid t', x^{t'}), \quad \forall\, (t, y^t), \forall\, \theta.$$

4. *For all $\mu_0 \in \Delta(\Theta)$, $F^\delta \geq_{cx} G^\delta$.*

Sets of discount factors + AS problems

Sets of discount factors

Use extreme points

1. (Weakly) decreasing discount factors:

$$\Delta_{\downarrow} := \{\delta \in \Delta(\{1,\dots,T\}) : \delta_1 \geq \delta_2 \geq \cdots \geq \delta_T\}$$

   $(1,0,\dots), (1/2,1/2,0,\dots), (1/3,1/3,1/3,0,\dots)$, etc.

2. All discount factors:

$$\Delta_{\updownarrow} := \Delta(\{1,\dots,T\})$$

   $(1,0,\dots), (0,1,0,\dots)$, etc.

3. Exponential discount factors in $(0,1)$.

$$\Delta_{\beta} := \left\{\delta^{\beta} :\ \delta^{\beta}_t = (1-\beta)\beta^{t-1},\ \beta \in (0,1)\right\}.$$

   Not convex!

   Result "for all sufficiently patient..."

Sets of discount factors

Proposition

f m.v. than g in discounted problems over $\Delta_{\downarrow} \iff \sum_{i=1}^{t} \frac{1}{t} f^i \geq_B \sum_{i=1}^{t} \frac{1}{t} g^i \ \forall\ t$.

Proposition

f m.v. than g in discounted problems over $\Delta_{\updownarrow} \iff f^t \geq_B g^t \ \forall\ t \iff f$ m.v. than g in AS problems.

Time Lotteries over Info

Time lotteries over info

Fix static experiments $\xi\colon \Theta \to \Delta(Z)$ and $\eta\colon \Theta \to \Delta(W)$. Let H and P be cdfs on $\{1, \dots, T\}$, with arrival times Y_H and Y_P.

Dynamic experiment $\mu^{H,\xi}$: draw $Y_H \sim H$ and $z \sim \xi(\cdot \mid \theta)$ independently, and set

$$x_t = \begin{cases} z, & t = Y_H, \\ \emptyset, & t \neq Y_H. \end{cases}$$

Define $\mu^{P,\eta}$ analogously. If $\xi = \eta$, write μ^H and μ^P.

Fix prior $\mu_0 \in \Delta(\Theta)$.

Notation. $\mu^{H,\xi} \succeq_\Delta \mu^{P,\eta}$ means: for every $\delta \in \Delta$ and every δ-discounted problem, $\mu^{H,\xi}$ gives weakly larger value.

Time lotteries over info: the key reduction

For a static experiment ζ, define its static value

$$W(\zeta; u) = \mathbb{E}\left[\max_{a\in A} \mathbb{E}[u(a,\theta) \mid s]\right] - \cancel{\max_{a\in A} \mathbb{E}[u(a,\theta)]}.$$

Set $w_\delta(y) := \sum_{t\geq y} \delta_t$.

If the signal arrives at time y, the gain is $w_\delta(y) W(\zeta; u)$: once the signal arrives, it can be used in every remaining period.

Proposition

Value of $\mu^{H,\xi} = \mathbb{E}_H\left[w_\delta(Y_H)\right] W(\xi; u)$, and value of $\mu^{P,\eta} = \mathbb{E}_P\left[w_\delta(Y_P)\right] W(\eta; u)$

So the timing question is about the shape of w_δ, not the shape of δ_t!

Back to example 2

For $T = 3$ and $(\delta_1, \delta_2, \delta_3) = \left(\frac{1}{2}, \frac{2}{5}, \frac{1}{10}\right)$,

$$w_\delta(1) = 1, \qquad w_\delta(2) = \frac{1}{2}, \qquad w_\delta(3) = \frac{1}{10}.$$

So if π is informative and $W(\pi; u) = \alpha > 0$,

$$\frac{1}{2} w_\delta(1)\alpha + \frac{1}{2} w_\delta(3)\alpha = \frac{11}{20}\alpha > \frac{1}{2}\alpha = w_\delta(2)\alpha.$$

Thus: 50/50 arrival at 1 or 3 beats sure arrival at 2.

By contrast, for a prize received at a random time,

$$\frac{1}{2}\delta_1 + \frac{1}{2}\delta_3 = \frac{3}{10} < \frac{2}{5} = \delta_2.$$

Same discount factor, opposite timing attitude.

Time lotteries over info: characterizations

Write $H(t) = \mathbb{P}(Y_H \leq t)$ and $P(t) = \mathbb{P}(Y_P \leq t)$.

Proposition

$\mu^{H,\xi} \succeq_\Delta \mu^{P,\eta}$ if and only if the corresp. ineq. below holds $\forall\ (A, u)$.

1. All discount factors $\Delta_\updownarrow$:

$$H(t)W(\xi;u) \geq P(t)W(\eta;u) \qquad \forall t.$$

2. Decreasing discount factors $\Delta_\downarrow$:

$$\sum_{t=1}^{m} H(t)W(\xi;u) \geq \sum_{t=1}^{m} P(t)W(\eta;u) \qquad \forall m.$$

3. Exponential discounting Δ_β, $T = \infty$:

$$\mathbb{E}[\beta^{Y_H}]W(\xi;u) \geq \mathbb{E}[\beta^{Y_P}]W(\eta;u) \qquad \forall \beta \in (0,1).$$

Same experiment, only timing differs

Assume $\xi = \eta$ and the common static experiment is nontrivial. Then the comparison depends only on the order of arrival times.

Corollary

$$\mu^H \geq_{\Delta_\updownarrow} \mu^P \iff Y_P \geq_1 Y_H \qquad \text{(FOSD)}$$
$$\mu^H \geq_{\Delta_\downarrow} \mu^P \iff Y_P \geq_2 Y_H \qquad \text{(increasing-concave order)}$$
$$\mu^H \geq_{\Delta_\beta} \mu^P \iff Y_P \geq_{pgf} Y_H.$$

Interpretation. Impatience $\Longrightarrow$ Time-risk love. Shape (beyond decreasing) does not matter!

probability-generating-function order

If the underlying experiments also differ

Define

w/ conventions $0/0 := 0$ and $a/0 := +\infty$ for $a > 0$

$$r_{\updownarrow} := \max_{1\le t\le T} \frac{P(t)}{H(t)}, \qquad r_{\downarrow} := \max_{1\le m\le T} \frac{\sum_{t=1}^{m} P(t)}{\sum_{t=1}^{m} H(t)},$$

and, for $T = \infty$,

$$r_{pgf} := \sup_{\beta\in(0,1)} \frac{\mathbb{E}[\beta^{Y_P}]}{\mathbb{E}[\beta^{Y_H}]}.$$

Write $\xi \succeq_r \eta$ if $W(\xi;u) \ge rW(\eta;u)$ for every decision problem (A,u). Then

$$\mu^{H,\xi} \succeq_{\Delta_{\updownarrow}} \mu^{P,\eta} \iff \xi \succeq_{r_{\updownarrow}} \eta,$$

and analogously for $\Delta_{\downarrow}$ and Δ_{β}.

If the relevant $r \le 1$, equivalent to ξ Blackwell-dominating the r-dilution of η.

Timing and informativeness separate multiplicatively.

Conclusion

In progress, r as a measure of comparative-VOI.

For discounted problems, aggregate then Blackwell.

Impatient DMs in discounted problems love risk.

Generalizations in the paper:

1. DM can (permanently) switch experiments after any history.
2. Actions affect info.
3. Actions affect discounting.
4. Time (In-)consistency?

Robust-to-outside-info analog is easy. AS and full class coincide.

Thank you!

Generalizations

Discounted problems – sequential comparisons

So far: assumed that the DM observes signals from either f or g (but not both).

As if the DM chooses *ex ante* whether to observe the X-signals or the Y-signals and *cannot switch*.

Strengthen our comparison: give the DM the opportunity to permanently switch from observing the X-signals to observing the Y-signals after any history x^t of X-signals.

Viz., want the DM to prefer observing signals from f not only at the *ex-ante* stage, but at all (on-path) histories.

Discounted problems – sequential comparisons

Consider the following example: two periods, two states L(ow) and H(high).

Signals are ℓ(ow) and h(igh). For $\theta = L, H$,

$$f_1(\ell|L) = f_1(h|H) = \frac{3}{4}, \qquad f_2(h|h,\theta) = f_2(\ell|\ell,\theta) = 1,$$

$$g_1(\ell|L) = g_1(h|H) = \frac{3}{4}, \qquad g_2(h|h,\theta) = g_2(\ell|\ell,\theta) = 1.$$

The two experiments seem identical and, indeed, have the same law.

As random processes, they may differ, however.

E.g., if they are cond. independent, after observing the first-period signal from either of them, the DM always has an incentive to switch.

If perfectly correlated, the DM never has an incentive to switch. $\Longrightarrow$ Need to model the interdependence.

Discounted problems – control of information

So far, the DM cannot influence/control the information he receives.

But...selling a product with unknown demand, experimentation/bandit problems, POMDP, etc.

Natural idea: To introduce actions in the experiments:

$$f_t : \Theta \times X^{t-1} \times A^{t-1} \to \Delta(X_t),$$
$$g_t : \Theta \times Y^{t-1} \times A^{t-1} \to \Delta(Y_t).$$

Problem: As the decision problem varies, so do the experiments!

Discounted problems – control of information

Our proposal: To have experiments depend on fixed covariates/controls K_t, which themselves are influenced/controlled by actions in A_t.

Conceptually, the same as having preferences over consequences, and actions influencing the consequences, as in mechanism design.

Controlled information: Two statistical experiments $f := (f_t)_t$ and $g := (g_t)_t$ to be compared, where

$$f_t : \Theta \times X^{t-1} \times K^{t-1} \to \Delta(X_t),$$
$$g_t : \Theta \times Y^{t-1} \times K^{t-1} \to \Delta(Y_t),$$

and K_t is the set of covariates at period t.

Decision problems: $(A_t, u_t, \kappa_t)_t$, with $\kappa_t : K^{t-1} \times A_t \to \Delta(K_t)$ the "control map."