Comparing Experiments in Discounted Problems

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2nd Workshop on Game Theory and Information, IHP
June 9, 2026

Dynamic decision problems

States: Θ, w/ generic element θ.

Time: $t = 1, \ldots, T$ $(T \leq \infty)$.

Dynamic information structures:

$$f : \Theta \to \Delta(X_1 \times \cdots \times X_T), \qquad \text{vs.} \qquad g : \Theta \to \Delta(Y_1 \times \cdots \times Y_T).$$

Dynamic decision problem: @ each t DM chooses $a_t \in A_t$ after observing past and current signals... *but not future ones.*

Utility: $u(a_1, \ldots, a_T, \theta)$.

All sets finite, except (possibly) # of pds.

Discounted problems

Additively-separable (AS) problem:

$$u(a_1,\dots,a_T,\theta) = \sum_{t=1}^{T} u_t(a_t,\theta),$$

for some cllxn. $(u_t : A_t \times \Theta \to \mathbb{R})_t$

Discounted problem: Common $A = A_t\ \forall t$, common $u : A \times \Theta \to \mathbb{R}$, &

$$u(a_1,\dots,a_T,\theta) = \sum_{t=1}^{T} \delta_t u(a_t,\theta), \quad \text{for some } (\delta_t)_t \in \Delta(\{1,\dots,T\}).$$

Misc. subclasses: all, decreasing, decreasing & convex, exponential, fixed.

Comparisons of info structures

Static problems (dynamic w/ $T = 1$).	Blackwell ('51, '53).
Dynamic problems.	Greenshtein ('96).
Discounted problems.	This talk.

Questions

1. Comparisons of info structures for discounted problems?

   1.1 Fixed discount.
   1.2 Set of discounts.
   1.3 Can switch experiments mid-problem.
   1.4 Actions affect state/info/future menus/discounting.

2. De Jarnette, Dillenberger, Gottlieb, Ortoleva ('20): attitudes toward timing risk. Here: timing risk over info arrival?

Roadmap

1. Brief review of Blackwell ('51, '53) & Greenshtein ('96).

2. Some examples.

3. Discounted problems: fixed discount factor.

4. Discounted problems: sets of discount factors + AS problems.

5. Risk preferences over info arrival.

Review

Review of Blackwell ('51, '53)

Single pd. dynamic problem: A finite set Θ, w/ generic element θ.

Information: Two statistical experiments $f\colon \Theta \to \Delta(X)$ and $g\colon \Theta \to \Delta(Y)$ to be compared.

Decision problem: (A, u), where A is the set of decisions and $u : A \times \Theta \to \mathbb{R}$ the utility/loss function.

Strategies: $\sigma\colon X \to \Delta(A)$ and $\tau\colon Y \to \Delta(A)$.

Distributions: $\mathbb{P}_{\theta,f,\sigma}$ and $\mathbb{P}_{\theta,g,\tau}$ are the induced distributions over signals and actions (given state θ).

Review of Blackwell ('51, '53)

Definition

The experiment f is more valuable than the experiment g if for all decision problems (A, u), for any strategy τ, there exists a strategy σ such that

$$\mathbb{E}_{\theta,g,\tau}[u(\mathbf{a},\theta)] \leq \mathbb{E}_{\theta,f,\sigma}[u(\mathbf{a},\theta)] \quad \forall\ \theta$$

Key: The information is fixed, but *all* decision problems are considered.

Remark: No prior is considered, hence "for all θ."

Given a prior $\mu_0 \in \Delta(\Theta)$, f induces (*ex-ante*) distribution over posteriors $F \in \Delta(\Delta(\Theta))$ w/ $\mathbb{E}_F(\mu) = \mu_0$.

Review of Blackwell ('51, '53)

Marginal dist. over actions: $(\mathrm{marg}_A \mathbb{P}_{\theta,g,\tau})(a) := \sum_{y \in Y} \mathbb{P}_{\theta,g,\tau}(y,a)$.

Theorem

T.f.a.e:

1. *The experiment f is more valuable than the experiment g.*
2. *For all decision problems (A,u), for all τ, there exists σ such that*

$$\mathrm{marg}_A \mathbb{P}_{\theta,g,\tau} = \mathrm{marg}_A \mathbb{P}_{\theta,f,\sigma}, \quad \forall\ \theta.$$

3. *Sufficiency: There exists a garbling $\gamma : X \to \Delta(Y)$ such that*

$$g(y|\theta) = \sum_x \gamma(y|x) f(x|\theta), \quad \forall\ y, \forall\ \theta$$

4. *For all $\mu_0 \in \Delta(\Theta)$, $F \geq_{cx} G$.*

Back to dynamics

Recall $f\colon \Theta \to \Delta(X_1 \times \cdots \times X_T)$ & utility $u(a_1, \ldots, a_T, \theta)$.

Write

$$X^t = X_1 \times \cdots \times X_t, \qquad \& \qquad A^t = A_1 \times \cdots \times A_t.$$

Strategy: a stochastic map

$$\alpha\colon X^T \to \Delta(A^T)$$

that is Adapted: for each t, the marginal

$$\alpha^t(a_1, \ldots, a_t \mid x_1, \ldots, x_T) = \sum_{a_{t+1}, \ldots, a_T} \alpha(a_1, \ldots, a_T \mid x_1, \ldots, x_T)$$

depends only on $(x_1, \ldots, x_t)$.

Key observation: strategies and garblings are the same type of object.

$$\Theta \xrightarrow{f} X^T \xrightarrow{\alpha} A^T.$$

Review of Greenshtein ('96)

Definition

The dynamic information structure f is *more informative* than g if for all dynamic decision problems and every adapted strategy τ for g, there exists an adapted strategy σ for f such that

$$\mathbb{E}_{\theta,g,\tau}[u(\mathbf{a}_1,\ldots,\mathbf{a}_T,\theta)] \leq \mathbb{E}_{\theta,f,\sigma}[u(\mathbf{a}_1,\ldots,\mathbf{a}_T,\theta)] \quad \forall\, \theta.$$

Key: mirrors Blackwell, only now the actions and signals arrive over time.

Implementable distributions over action histories:

$$\Lambda_f(A_1,\ldots,A_T) = \{\alpha \circ f : \ \alpha \text{ adapted}\}.$$

Review of Greenshtein ('96). Also De Oliveira ('17)

Theorem

T.f.a.e:

1. *f is more informative than g in every dynamic decision problem.*
2. *For every action space $(A_t)_t$,*

$$\Lambda_f(A_1,\ldots,A_T) \supseteq \Lambda_g(A_1,\ldots,A_T).$$

3. *There exists an adapted garbling*

$$\gamma\colon X^T \to \Delta(Y^T)$$

s.t. $g = \gamma \circ f$.

4. *Sequences of distributions over posteriors?*

Blackwell's siren call

For each t, view dynamic experiment f as static experiment $f^t \to \Delta(X^t)$.

$$\text{Greenshtein} \quad \overset{?}{\underset{?}{\equiv}} \quad f^t \succeq_B g^t \quad \forall\ t$$

Examples

Example 1: a (Wald) stopping problem

Two states and two actions:

$$\theta \in \{0,1\}, \qquad a \in \{0,1\}, \qquad \& \qquad \mu_0 = \mathbb{P}(\theta = 1).$$

Two conditionally-independent binary signals: Weak and Strong:

	$\mathbb{P}(W = 1 \mid \theta)$	$\mathbb{P}(S = 1 \mid \theta)$
$\theta = 0$	$1/3$	$1/4$
$\theta = 1$	$2/3$	$3/4$

Two sequential experiments:

$$f = (W, S), \qquad \& \qquad g = (S, W).$$

The first signal is "free." Observing the second signal costs $c = 1/100$.

Example 1: utilities

Payoffs:

	$\theta = 0$	$\theta = 1$
$a = 0$	1	0.9
$a = 1$	0	1

Plus: if the second signal is bought, subtract $c = 1/100$.

$$v(q) = \max\left\{1 - \frac{q}{10}, q\right\}.$$

Static cutoff:

$$a = 1 \quad \Longleftrightarrow \quad q \geq \frac{10}{11}.$$

Example 1: decisions

Set $\mu_0 = \mathbb{P}(\theta = 1) = \frac{4}{5}$.

Supports are

$$f: \quad \frac{4}{5} \to \left\{ \frac{2}{3}, \overset{w.p.\ 12/20}{\frac{8}{9}} \right\} \to \left\{ \frac{2}{5}, \frac{8}{11}, \frac{6}{7}, \frac{24}{25} \right\},$$

$$g: \quad \frac{4}{5} \to \left\{ \frac{4}{7}, \underset{w.p.\ 13/20}{\frac{12}{13}} \right\} \to \left\{ \frac{2}{5}, \frac{8}{11}, \frac{6}{7}, \frac{24}{25} \right\}.$$

Can write "Wald order..." strictly weaker than Greenshtein.

Example 1.5: discounting

Previous decision problem: not a discounted problem.

Not AS.

Take arbitrary AS problem; for $t = 1, 2$ and $p \in [0, 1]$ write $V_t(p) := \max_{a_t} \mathbb{E}_p u_t(a_t, \theta)$.

$$\mathbb{E}_{G_1} V_1 \geq \mathbb{E}_{F_1} V_1 \ \& \ \mathbb{E}_{G_2} V_2 \geq \mathbb{E}_{F_2} V_2 \quad \Longrightarrow \quad g \text{ better for AS problems!}$$

$\Longrightarrow$ g superior to f for discounted problems (all varieties).

The Main Result

Discounted problems – reminder

States: A set Θ, w/ generic element θ.

Horizon: $T \leq +\infty$ periods, w/ generic element t.

Notation. $X^t := X_1 \times \cdots \times X_t$, etc.

Information: Two statistical experiments $f := (f_t)_t$ and $g := (g_t)_t$ w/

$$f_t : \Theta \times X^{t-1} \to \Delta(X_t), \qquad \& \qquad g_t : \Theta \times Y^{t-1} \to \Delta(Y_t).$$

AS Decision problems: $(A_t, u_t)_t$, w/ A_t is the set of decisions @ t, $u_t : A_t \times \Theta \to \mathbb{R}$ the utility @ t.

AS problem: DM evaluates $(a_t)_t$ via $\sum_t u_t(a_t, \theta)$.

Reminder

Discounted problem: $(A_t, u_t) = (A, u)$ for all t, & DM evaluates $(a_t)_t$ as

$$\sum_t \delta_t u(a_t, \theta), \qquad \text{for some } (\delta_t)_t \in \Delta(\{1, \ldots, T\}).$$

Strategies:

$$\sigma_t : X^t \times A^{t-1} \to \Delta(A_t) \qquad \& \quad \tau_t : Y^t \times A^{t-1} \to \Delta(A_t).$$

Distributions: $\mathbb{P}_{\theta,f,\sigma}$ and $\mathbb{P}_{\theta,g,\tau}$ are the induced distributions over signals and actions (given θ).

Comparing experiments

Definition

Let δ be the discount factor. The statistical experiment f is *more valuable* than g in discounted problems, if for all decision problems (A, u), for any strategy τ, there exists strategy σ such

$$\mathbb{E}_{\theta,g,\tau}\left[\sum_t \delta_t u(\mathbf{a}_t, \theta)\right] \leq \mathbb{E}_{\theta,f,\sigma}\left[\sum_t \delta_t u(\mathbf{a}_t, \theta)\right], \quad \forall\ \theta$$

Key: Discount factor is fixed. Later: sets.

Aim: Derive the analog of Blackwell/Greenshtein.

Marginals

Time-weighted marginal dist. over actions:
$(\sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,g,\tau})(a) := \sum_t \sum_{y^t \in Y^t} \delta_t \mathbb{P}_{\theta,g,\tau}(y^t, a)$
Note:

$$\mathbb{E}_{\theta,f,\sigma}\left[\sum_t \delta_t u(\mathbf{a}_t, \theta)\right] = \sum_a u(a,\theta) \underbrace{\left[\sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,f,\sigma}(a)\right]}_{\text{discounted prob. of "}a\text{"}}.$$

Hence, as in static problems ((1) $\Longleftrightarrow$ (2) in Blackwell):

Lemma. f is more valuable than g in all discounted problems if, and only if, for all τ, there exists σ, such that

$$\sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,g,\tau} = \sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,f,\sigma}, \quad \forall\, \theta.$$

Marginals to garblings

Now want analog between (2) $\iff$ (3) in Blackwell, *viz.*, via garblings.

From the lemma, restrict attention to ensuring that:

$$\sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,g,\tau} = \sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,f,\sigma},$$

i.e., matching the discounted probabilities of decisions.

Garblings

We have:

$$\sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,f,\sigma}(a) = \sum_t \sum_{x^t \in X^t} \delta_t \mathbb{P}_{\theta,f,\sigma}(x^t) \mathbb{P}_{\theta,f,\sigma}(a|x^t).$$

Moreover, $\mathbb{P}_{\theta,f,\sigma}(x^t)$ does not depend on σ and is equal to:

$$f^t(x^t|\theta) := f_1(x_1|\theta) \times \cdots \times f_t(x_t|x^{t-1}, \theta).$$

So, we can rewrite the above in the form:

$$\sum_t \sum_{x^t \in X^t} \underbrace{\delta_t f^t(x^t|\theta)}_{\text{info}} \underbrace{\overline{\sigma}(a|t, x^t)}_{\text{strategy}}.$$

Punchline: Back to a static problem! Signal is now (t, x^t) w/ prob. $\delta_t f^t(x^t|\theta)$.

δ-sufficiency

Similarly,

$$\sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,g,\tau}(a) = \sum_t \sum_{y^t \in Y^t} \delta_t g^t(y^t|\theta)\overline{\tau}(a|t, y^t).$$

From (2) $\iff$ (3) in Blackwell, need the existence of a garbling γ satisfying

$$\delta_t g^t(y^t|\theta) = \sum_{t',x^{t'}} \delta_{t'} f^{t'}(x^{t'}|\theta)\gamma(\underbrace{t, y^t}_{\text{"simulations"}} |t', x^{t'}),$$

for all θ, for all (t, y^t).

Call this condition δ-sufficiency.

Intuition. It does not matter when "a" is chosen, as long as the averages match.

Ex: info today vs. info tomorrow

Two periods, equally discounted $(\delta_1, \delta_2) = (1/2, 1/2)$.

- f: no info in period 1, but full info in period 2.
- g: some info in period 1, but no further info in period 2.

$\theta \backslash x$	x_0	x_1
θ_0	1	0
θ_1	0	1

Table: f

$\theta \backslash y$	y_0	y_1
θ_0	7/12	5/12
θ_1	5/12	7/12

Table: g

Is pd 2. f is more informative than g in static problems, but is it "sufficiently more informative"?

Ex: info today vs. info tomorrow

δ-sufficiency amounts to comparing a modified version of f with g, where

$\theta \backslash x$	x_0	x_1	$\emptyset$
θ_0	1/2	0	1/2
θ_1	0	1/2	1/2

Table: the modified f

Modified f: with prob. 1/2, no info; with prob. 1/2, full info.

Answer: The "modified f" is sufficient for g, hence f is δ-sufficient for g.

Alternative approach

Given $\mu_0 \in \Delta(\Theta)$, sequences of distributions over posteriors:

$$(F_t)_t \qquad \text{vs.} \qquad (G_t)_t.$$

Define

$$F^\delta := \sum_{t=1}^{T} \delta_t F_t \quad \& \quad G^\delta := \sum_{t=1}^{T} \delta_t G_t.$$

Proposition

f m.v. than g in discounted problems w/ factor δ if and only if $F^\delta \geq_{cx} G^\delta \ \forall \ \mu_0$.

Alternative approach

Proposition

f m.v. than g in discounted problems w/ factor δ if and only if $F^{\delta} \geq_{cx} G^{\delta}$ $\forall$ μ_0.

proof.

$$\sum_{t=1}^{T} \delta_t \mathbb{E}_{F_t} V(\mu) \geq \sum_{t=1}^{T} \delta_t \mathbb{E}_{G_t} V(\mu) \; \forall \text{ convex } V \iff \mathbb{E}_{\sum_{t=1}^{T} \delta_t F_t} V(\mu) \geq \mathbb{E}_{\sum_{t=1}^{T} \delta_t G_t} V(\mu)$$

$$\iff \mathbb{E}_{F^{\delta}} V(\mu) \geq \mathbb{E}_{G^{\delta}} V(\mu).$$

Discounted problems – the basics

Theorem

T.f.a.e:

1. *f m.v. than g in discounted problems w/ factor δ.*
2. *For all decision problems (A, u), for all strategies τ, there exists a strategy σ s.t.*

$$\sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,g,\tau} = \sum_t \delta_t \mathrm{marg}_{A_t} \mathbb{P}_{\theta,f,\sigma}, \quad \forall\, \theta.$$

3. *δ-sufficiency: There exist garbling γ s.t.*

$$\delta_t g^t(y^t|\theta) = \sum_{t'} \sum_{x^{t'} \in X^{t'}} \delta_{t'} f^{t'}(x^{t'}|\theta)\gamma(t, y^t \mid t', x^{t'}), \quad \forall\, (t, y^t),\ \forall\, \theta.$$

4. *For all $\mu_0 \in \Delta(\Theta)$, $F^\delta \geq_{cx} G^\delta$.*

Sets of discount factors + AS problems

Sets of discount factors

$\{\delta\colon f \text{ is } \delta\text{-sufficient for } g\}$ is convex. $\implies$ Use extreme points.

1. (Weakly) decreasing discount factors:

$$\Delta_{\downarrow} := \{\delta \in \Delta(\{1,\dots,T\}) : \delta_1 \geq \delta_2 \geq \cdots \geq \delta_T\}$$

   $(1,0,\dots)$, $(1/2,1/2,0,\dots)$, $(1/3,1/3,1/3,0,\dots)$, etc.

2. All discount factors:

$$\Delta_{\updownarrow} := \Delta(\{1,\dots,T\})$$

   $(1,0,\dots)$, $(0,1,0,\dots)$, etc.

3. Exponential discount factors in $(0,1)$ (& $T=\infty$).

$$\Delta_{\beta} := \left\{\delta^{\beta}\colon\ \delta_t^{\beta} = (1-\beta)\beta^{t-1},\ \beta \in (0,1)\right\}.$$

Not convex! Result "for all sufficiently patient..."

Sets of discount factors

Proposition

f m.v. than g in discounted problems over $\Delta_{\downarrow} \iff \sum_{i=1}^{t} \frac{1}{t} f^i \geq_B \sum_{i=1}^{t} \frac{1}{t} g^i \ \forall\ t$.

Proposition

f m.v. than g in discounted problems over $\Delta_{\updownarrow} \iff f^t \geq_B g^t \ \forall\ t$.

Proposition

f m.v. than g in AS problems $\iff f^t \geq_B g^t \ \forall\ t$.

Time Lotteries over Info

Time lotteries over info

Fix static experiments $\xi\colon \Theta \to \Delta(Z)$ and $\eta\colon \Theta \to \Delta(W)$. Let H and P be cdfs on $\{1, \ldots, T\}$, with arrival times Y_H and Y_P.

Dynamic experiment $\mu^{H,\xi}$: draw $Y_H \sim H$ and $z \sim \xi(\cdot \mid \theta)$ independently, and set

$$x_t = \begin{cases} z, & t = Y_H, \\ \emptyset, & t \neq Y_H. \end{cases}$$

Define $\mu^{P,\eta}$ analogously. If $\xi = \eta$, write μ^H and μ^P.

Fix prior $\mu_0 \in \Delta(\Theta)$.

Notation. $\mu^{H,\xi} \succeq_\Delta \mu^{P,\eta}$ means: for every $\delta \in \Delta$ and every δ-discounted problem, $\mu^{H,\xi}$ gives weakly larger value.

Time lotteries over info: the key reduction

For a static experiment ζ, define its static value

$$W(\zeta; u) = \mathbb{E}\left[\max_{a \in A} \mathbb{E}[u(a,\theta) \mid s]\right] - \max_{a \in A} \mathbb{E}[u(a,\theta)].$$

Set $w_\delta(y) := \sum_{t \geq y} \delta_t$.

If the signal arrives at time y, the gain is $w_\delta(y) W(\zeta; u)$: once the signal arrives, it can be used in every remaining period.

Proposition

Value of $\mu^{H,\xi} = \mathbb{E}_H[w_\delta(Y_H)]\, W(\xi; u)$, and value of $\mu^{P,\eta} = \mathbb{E}_P[w_\delta(Y_P)]\, W(\eta; u)$

So the timing question is about the shape of w_δ, not the shape of δ_t!

Time lotteries over info: characterizations

Write $H(t) = \mathbb{P}(Y_H \leq t)$ and $P(t) = \mathbb{P}(Y_P \leq t)$.

Proposition

$\mu^{H,\xi} \succeq_\Delta \mu^{P,\eta}$ if and only if the corresp. ineq. below holds $\forall\ (A, u)$.

1. All discount factors $\Delta_{\updownarrow}$:

$$H(t)W(\xi; u) \geq P(t)W(\eta; u) \qquad \forall t.$$

2. Decreasing discount factors $\Delta_{\downarrow}$:

$$\sum_{t=1}^{m} H(t)W(\xi; u) \geq \sum_{t=1}^{m} P(t)W(\eta; u) \qquad \forall m.$$

3. Exponential discounting Δ_β, $T = \infty$:

$$\mathbb{E}[\beta^{Y_H}]W(\xi; u) \geq \mathbb{E}[\beta^{Y_P}]W(\eta; u) \qquad \forall \beta \in (0, 1).$$

Same experiment, only timing differs

Assume $\xi = \eta$ and the common static experiment is nontrivial. Then the comparison depends only on the order of arrival times.

Corollary

$$\mu^H \succeq_{\Delta_\updownarrow} \mu^P \iff Y_P \geq_1 Y_H \qquad \text{(FOSD)}$$

$$\mu^H \succeq_{\Delta_\downarrow} \mu^P \iff Y_P \geq_2 Y_H \qquad \text{(increasing-concave order)}$$

$$\mu^H \succeq_{\Delta_\beta} \mu^P \iff Y_P \geq_{pgf} Y_H.$$

Interpretation. Impatience $\implies$ Time-risk love. Shape (beyond decreasing) does not matter!

probability-generating-function order

If the underlying experiments also differ

Define

w/ conventions $0/0 := 0$ and $a/0 := +\infty$ for $a > 0$

$$r_{\updownarrow} := \max_{1\le t\le T} \frac{P(t)}{H(t)}, \qquad r_{\downarrow} := \max_{1\le m\le T} \frac{\sum_{t=1}^{m} P(t)}{\sum_{t=1}^{m} H(t)},$$

and, for $T = \infty$,

$$r_{pgf} := \sup_{\beta\in(0,1)} \frac{\mathbb{E}[\beta^{Y_P}]}{\mathbb{E}[\beta^{Y_H}]}.$$

Write $\xi \succeq_r \eta$ if $W(\xi;u) \ge rW(\eta;u)$ for every decision problem (A,u). Then

$$\mu^{H,\xi} \succeq_{\Delta_{\updownarrow}} \mu^{P,\eta} \iff \xi \succeq_{r_{\updownarrow}} \eta,$$

and analogously for $\Delta_{\downarrow}$ and Δ_{β}.

If the relevant $r \le 1$, equivalent to ξ Blackwell-dominating the r-dilution of η.

Timing and informativeness separate multiplicatively.

Doeblin coeff., Kim & Obara

Conclusion

For discounted problems, aggregate then Blackwell.

Impatient DMs in discounted problems love risk.

Generalizations in the paper:

1. DM can (permanently) switch experiments after any history.
2. Actions affect info.
3. Actions affect future menus.
4. Actions affect discounting.

Robust-to-outside-info analog is easy. AS and full class coincide.

Aux. note: r as a measure of comparative-VOI.

Thank you!

Generalizations

Discounted problems – sequential comparisons

So far: assumed that the DM observes signals from either f or g (but not both).

As if the DM chooses *ex ante* whether to observe the X-signals or the Y-signals and *cannot switch*.

Strengthen our comparison: give the DM the opportunity to permanently switch from observing the X-signals to observing the Y-signals after any history x^t of X-signals.

Viz., want the DM to prefer observing signals from f not only at the *ex-ante* stage, but at all (on-path) histories.

Discounted problems – sequential comparisons

Consider the following example: two periods, two states L(ow) and H(high).

Signals are ℓ(ow) and h(igh). For $\theta = L, H$,

$$f_1(\ell|L) = f_1(h|H) = \frac{3}{4}, \qquad f_2(h|h,\theta) = f_2(\ell|\ell,\theta) = 1,$$

$$g_1(\ell|L) = g_1(h|H) = \frac{3}{4}, \qquad g_2(h|h,\theta) = g_2(\ell|\ell,\theta) = 1.$$

The two experiments seem identical and, indeed, have the same law.

As random processes, they may differ, however.

E.g., if they are cond. independent, after observing the first-period signal from either of them, the DM always has an incentive to switch.

If perfectly correlated, the DM never has an incentive to switch. $\Longrightarrow$ Need to model the interdependence.

Discounted problems – control of information

So far, the DM cannot influence/control the information he receives.

But...selling a product with unknown demand, experimentation/bandit problems, POMDP, etc.

Natural idea: To introduce actions in the experiments:

$$f_t : \Theta \times X^{t-1} \times A^{t-1} \to \Delta(X_t),$$
$$g_t : \Theta \times Y^{t-1} \times A^{t-1} \to \Delta(Y_t).$$

Problem: As the decision problem varies, so do the experiments!

Discounted problems – control of information

Our proposal: To have experiments depend on fixed covariates/controls K_t, which themselves are influenced/controlled by actions in A_t.

Conceptually, the same as having preferences over consequences, and actions influencing the consequences, as in mechanism design.

Controlled information: Two statistical experiments $f := (f_t)_t$ and $g := (g_t)_t$ to be compared, where

$$f_t : \Theta \times X^{t-1} \times K^{t-1} \to \Delta(X_t),$$
$$g_t : \Theta \times Y^{t-1} \times K^{t-1} \to \Delta(Y_t),$$

and K_t is the set of covariates at period t.

Decision problems: $(A_t, u_t, \kappa_t)_t$, with $\kappa_t : K^{t-1} \times A_t \to \Delta(K_t)$ the "control map."